Classical fractional discrete time crystals

The goal of this project is to investigate the dynamics of a classical time crystal. In particular we want to observe fractional spontaneous symmetry breaking in the prethermal plateau of an all-to-all coupled spin model with Hamiltonian

$$H(t) = \frac{J}{N} \sum_{i,j=1}^{N} \sigma_i^z \sigma_j^z - \pi h[1 + \sin(2\pi t)] \sum_j \sigma_j^x \tag{1}$$

where σ_j^α denote the Pauli matrices, J sets the strength of the Ising interaction, and h is the strength of the spin flip field. This model is known as the Lipkin-Meshkov-Glick (LMG) model, and can be mapped to a system of two interacting bosons ($\psi_{\uparrow/\downarrow}$) whose dynamics can be investigated. The project is based on the paper *Higher-order and fractional discrete time crystals in clean long-range interacting systems* by Pizzi *et al.*, Nature Communications volume 12, Article number: 2341 (2021). This project is numerical with a small analytical component, and requires writing a custom code.

- Derive the EOM given in Eq. (2) of the above paper from the LMG model (see supplemental material to paper). Show that the spin magnetization m is given by $m = |\psi_\uparrow|^2 - |\psi_\downarrow|^2$.
- Write a code to numerically solve the corresponding Gross-Pitaevskii equation (GPE), i.e. Eq. (2). Plot the dynamics of the magnetization $m(t)$ and find signatures of a DTC.
- Reproduce the figure below (Fig. 1 from the paper).

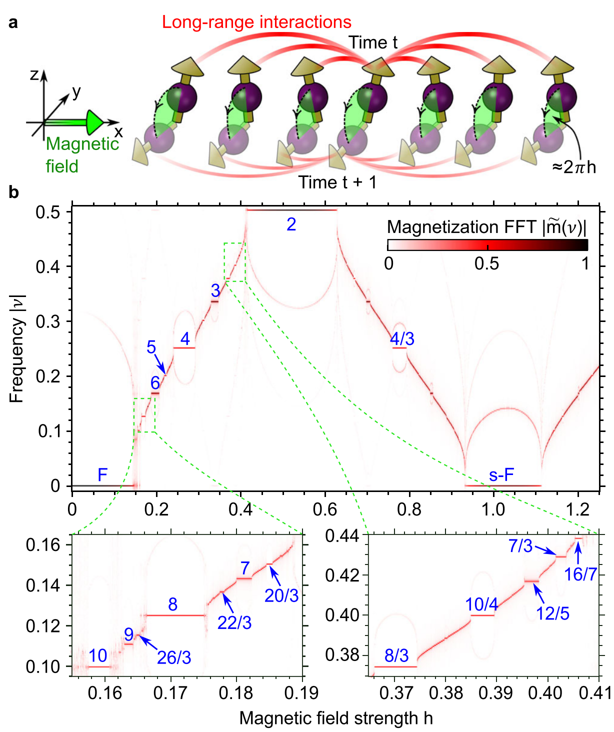

Figure 1: Signatures of a fractional classical discrete time crystal. (For more details, see reference.)

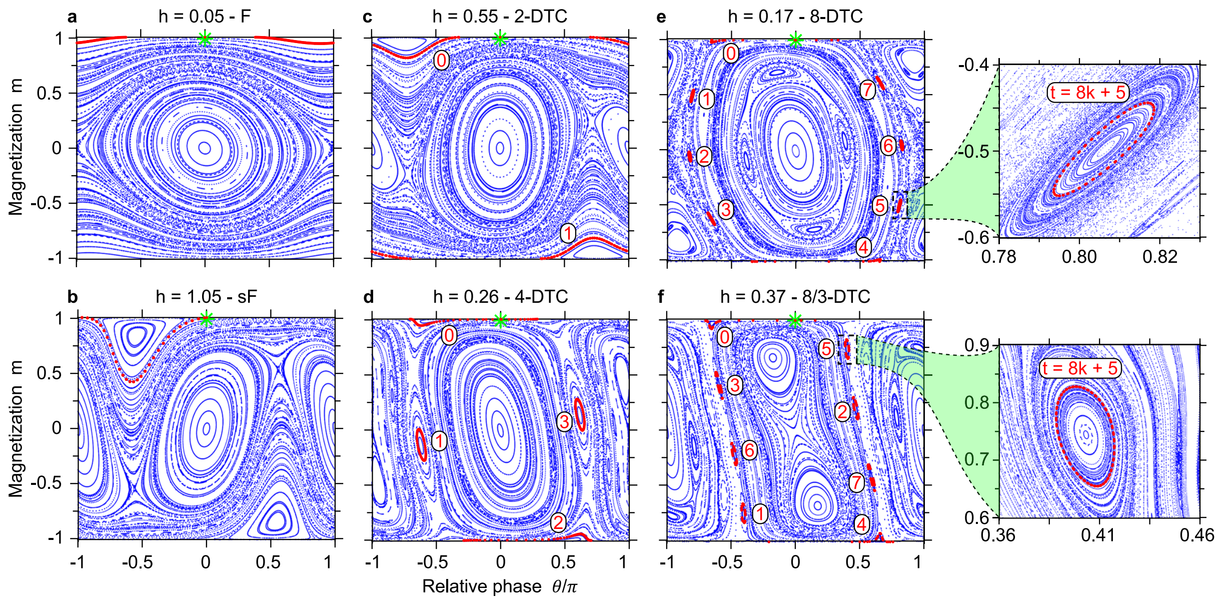

Figure 2: Poincare maps of the semiclassical dynamics, illustrating the emergence of 'islands' responsible for the DTC dynamics. (For more details, see reference.)

- Investigate the structure of the phase space portrait and reproduce Fig 2 from the paper.
- Optional goal: Add small temporal perturbations to the periodic drive: $\pi h[1 + \sin(2\pi t)] \rightarrow \pi h[1 + \sin(2\pi t(1 + \delta))]$ where δ is a random number drawn from a normal distribution with zero mean and variance you can control. Investigate how the fractional response reacts as the value of the noise δ is increased.