Operator spreading in classical and quantum dynamics

In this project we want to constrast and compare information spreading in classical cellular automata with operator spreading in quantum circuits. This project is based on the papers *Butterfly effect and spatial structure of information spreading in a chaotic cellular automaton*, Liu *et al.*, Phys. Rev. B **103**, 094109 (2021) and *Information scrambling in quantum circuits*, Google Quantum AI, Science **374**, 1479–1483 (2021). This project combines analytical and numerical approaches.

- Classical cellular automata: In cellular automata we consider a lattice of sites, where each site can take a discrete value 0 or 1. We can consider discrete time dynamics where the values on all sites are updated according to classical (possibly probabilistic) local update rules. A classical OTOC can then be defined as the local distance between two copies of the same system which differ by only a local perturbation of the initial conditions. Reproduce Fig. 1 of the first paper, illustrating how information spreads with a finite (butterfly) velocity in lattice systems.

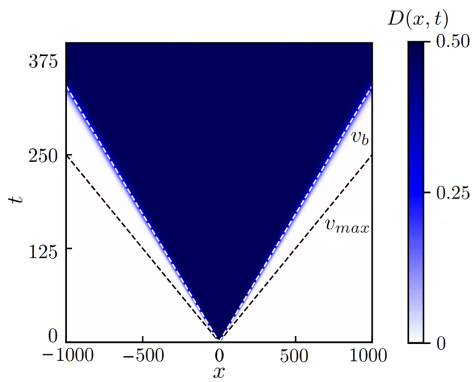

Figure 1: Classical OTOC for a Kauffman cellular automaton. The information spreads with a butterfly velocity v_b that is smaller than the maximal allowed butterfly velocity. See reference for details.

- Quantum circuits: In the second paper the OTOC is experimentally measured and contrasted for two classes of circuit dynamics in Fig. 2, namely for iSWAP dynamics and $\sqrt{\text{iSWAP}}$ dynamics. The former leads to a maximal butterfly velocity, whereas the latter results in a slower spreading of information and a nonmaximal butterfly velocity. It is possible to construct cellular automata that exhibit the same qualitative dynamics. Numerically calculate the classical OTOC for a circuit model where every gate is a randomly selected injective classical cellular automaton, i.e. the local update rules map $\{00, 01, 10, 11\}$ to a random permutation of $\{00, 01, 10, 11\}$. Derive the classical Markov process that corresponds to this update rules and show that it has a nonmaximal butterfly velocity, as for the $\sqrt{\text{iSWAP}}$ dynamics. Restrict the classical cellular automata to be randomly sampled from

$$\begin{aligned}\{00,01,10,11\} \quad \rightarrow \quad & \{00,10,01,11\},\{00,10,11,01\},\{00,11,01,10\},\{01,10,00,11\},\\ & \{01,11,00,10\},\{01,11,10,00\},\{10,00,01,11\},\{10,00,11,01\},\\ & \{10,01,11,00\},\{11,00,10,01\},\{11,01,00,10\},\{11,01,10,00\} \qquad (1)\end{aligned}$$

  Compare the corresponding OTOC dynamics and Markov dynamics, showing that these models behave qualitatively similar to the iSWAP dynamics.

- Optional goals: Compare the dynamics on a 2D lattice similar to Google's Sycamore chip. Consider different random realization of the cellular automata and derive the corresponding butterfly velocity. More advanced: Consider Sec. IV. on Markov population dynamics from the supplementary material to the second reference and implement these Markovian dynamics on a one-dimensional lattice. How do these differ from the simplified dynamics for the cellular automata?