Counterdiabatic control in classical and quantum two-spin systems

In this project we want to see how tools from quantum counterdiabatic control can be applied in a classical setting. The goal is to first derive and simulate a counterdiabatic protocol in a quantum two-spin model, and subsequently apply this protocol in a classical setting and check its effectiveness. This project is based on the papers *Geometry and non-adiabatic response in quantum and classical systems*, Kolodrubetz *et al.*, Phys. Rep. **697**, 1–87 (2017), specifically Section 2.5 and 3.5, and *Floquet-Engineering Counterdiabatic Protocols in Quantum Many-Body Systems*, Claeys *et al.*, Phys. Rev. Lett. **123**, 090602 (2019). This project has both an analytical and a numerical component.

Specifically, we consider a two-spin system

$$\hat{H}(t) = J(S_1^x S_2^x + S_1^y S_2^y) + h(t)(S_1^z + S_2^z), \tag{1}$$

and vary $h(t)$ from a large initial value (where the ground state is a product state) to 0 (where the ground state is a Bell state). The aim is to extend the quantum counterdiabatic protocol to a classical setup.

- Explicitly construct the counterdiabatic term using the approach from the second reference. Simulate the dynamics of the quantum system using QuSpin and verify that exact state preparation is possible using counterdiabatic driving. Reproduce the UA and CD results from Fig. 2 in this reference.

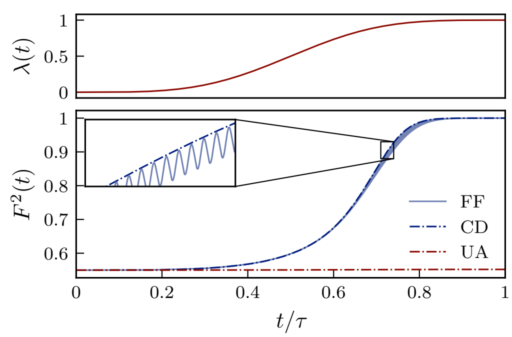

Figure 1: Fidelity $F^2(t) = |\langle\psi_0(\lambda(t))|\psi(t)\rangle|^2$ comparing the time-evolved state to the instantaneous ground state $|\psi_0\rangle$ of the Hamiltonian with $h(t) = 1 - \lambda(t)$, comparing the unassisted protocol (UA), the counterdiabatic procotol (CD) and a Floquet-engineered realization of the CD protocol (FF). See reference for details.

- Consider the same functional form of the counterdiabatic term, now in terms of classical spins, and rederive the prefactor using the variational approach for classical spins. Simulate the same dynamics as in the quantum case for an ensemble of classical trajectories by numerically integrating the classical equations of motion for a Gaussian distribution of initial conditions (centered on the fully polarized state). Compare the unassisted dynamics to the counterdiabatic dynamics by visualizing the phase space distributions.

- Optional goals: Include additional control terms and check how these modify the counterdiabatic drive. Numerically calculate the classical mutual information between the phase space distributions for the two spins as a function of time, and check the effect of the counterdiabatic term.