

Frequency scaling of photo-induced tunneling

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Abstract. – The DC current-voltage characteristics induced by a driving electric field with frequency Ω of a one-dimensional electron channel with a tunnel barrier is calculated. Electron-electron interaction of finite range is taken into account. For intermediate interaction strengths, the non-linear differential conductance shows cusp-like minima at bias voltages $m\hbar\Omega/e$ (m integer) that are a consequence of the finite non-zero range of the interaction but are independent of the shape of the driving electric field. However, the frequency-scaling of the photo-induced current shows a cross-over between Ω^{-1} and Ω^{-2} , and depends on the spatial shape of the driving field and the range of the interaction.

The influence of a monochromatic driving field on the transport through tunnel barriers has become an extensively studied subject. A paradigmatic result has been obtained more than three decades ago by Tien and Gordon for a superconducting tunnel junction [1]. Similar approaches have been used to study photo-assisted effects in the tunneling through quantum point contacts [2] and driven quantum wells [3]. Basically, the frequency of the driving field has been shown to produce sidebands due to the non-linearity of the current-voltage characteristics. In all of these works, interaction between the electrons has been neglected. Recently, the influence of electron-electron interaction on non-linear tunneling transport has been studied in the regions of charging, interaction [4-6] and strong correlations [7-10]. In the mean-field approximation, renormalisation of the driving field has been studied [11]. Also experimentally, in view of the realization of the quantum dot current turnstile device [12], photo-induced tunneling through quantum dots has been often addressed [13-16].

In this paper, we present physically striking results from evaluating the general theory developed earlier, applied to driven non-linear transport through a tunnel barrier in a one-dimensional (1D) system of interacting electrons. We report a novel frequency-locking effect,

which is signature of the systems coherent, *strongly* correlated electron states. It is characteristic of the finite, non-zero range of the interaction but does not depend on the exact shape of the driving electric field.

By starting from the assumption that the current is driven by a local electric field of arbitrary shape, with a DC-component $E_0(x)$ and an additional monochromatic component with frequency Ω , $E(x, t) = E_0(x) + E_1(x) \cos \Omega t$, we find an extremely rich behaviour of the photo-induced current-voltage characteristic $I(V_0; V_1, \Omega)$ as a function of various parameters, like the range and strength of the interaction and the spatial shape of the applied field (V_0, V_1 voltages corresponding to E_0, E_1 , respectively). i) We first re-derive for finite-range interaction the non-trivial general expression for $I(V_0; V_1, \Omega)$ in terms of the (non-linear) DC current-voltage characteristic $I_0(V_0) = I(V_0; V_1 = 0, \Omega = 0)$ [8, 17, 18]

$$I(V_0; V_1, \Omega) = \sum_{n=-\infty}^{\infty} J_n^2(|z|) I_0 \left(V_0 + n \frac{\hbar \Omega}{e} \right). \quad (1)$$

It generalises the Tien-Gordon result to strongly correlated tunneling objects and arbitrary shapes of the driving fields. ii) We show the existence of pronounced, cusp-like minima in the differential conductance dI/dV_0 at integer ratios of $eV_0/\hbar\Omega$ if the dimensionless interaction parameter $g \geq 2/3$, independent of how exactly the driving voltage drops. These structures are due to sidebands that are induced by the non-linearity of the DC current-voltage characteristics [17, 18], and are *independent* of the shape of the driving electric field. Finally, iii) we predict a cross-over between frequency scaling $\propto \Omega^{-1}$ and $\propto \Omega^{-2}$ for delta-function-like and spatially constant driving fields, respectively. In the cross-over region the frequency scaling is non-universal and *reflects* the ranges of both, the electron-electron interaction and the range of electric field.

It is well known that the interaction-induced renormalisation of the externally applied field influences the current in the frequency and time domains [9-11]. Our results indicate that for 1D correlated electrons, as long as the driving field is monochromatic, the Tien-Gordon formula remains valid, but with a general argument of the Bessel functions. The latter is given by the modulus of the complex quantity

$$z = \frac{e}{\hbar \Omega} \int_{-\infty}^{\infty} dx E_1(x) r(x, \Omega), \quad (2)$$

and contains the spatial shape of the AC component of the driving field, and the interaction range via the AC conductivity $\sigma(x, \Omega)$ without tunnel barrier, $r(x, \Omega) = \sigma(x, \Omega)/\sigma(0, \Omega)$.

In order to study the effect of finite-range interactions and a space-dependent electric field on non-linear AC transport, we use the 1D Luttinger liquid model [19-21]. The Hamiltonian for a spinless Luttinger liquid with an impurity and subject to a time-dependent electric field is $H = H_0 + H_t + H_{ac}$, where H_0 is the Hamiltonian describing charge density excitations with a dispersion $\omega(k) = v_F |k| [1 + \hat{V}_{ee}(k)/\hbar \pi v_F]^{1/2}$ (v_F Fermi velocity). It reflects the Fourier transform of the interaction potential [22]. We assume a 3D screened Coulomb interaction of range α^{-1} projected onto a quantum wire of the diameter d . The dimensionless interaction parameter of the model is $g = [1 + \hat{V}_{ee}(k=0)/\hbar \pi v_F]^{-1/2}$. For $\alpha^{-1} \gg d$, the interaction decays exponentially and one obtains the ‘‘Luttinger limit’’, $V_{ee}(x) \propto \alpha e^{-\alpha|x|}$ [22].

The tunneling barrier of height U_t is assumed to be localised at $x = 0$ [7],

$$H_t = U_t \cos [2\sqrt{\pi} \vartheta(x = 0)], \quad (3)$$

with the phase variable $\vartheta(x)$ giving the charge density fluctuations in the limit of long wave-

lengths $\rho(x) = k_F/\pi + \partial_x \vartheta(x)/\sqrt{\pi}$. The coupling to the driving field is

$$H_{ac} = e \int_{-\infty}^{\infty} dx \rho(x) V(x, t). \quad (4)$$

The voltage $V(x, t)$ is related to the field by $E(x, t) = -\partial_x V(x, t)$. For the space dependence of the electric field we assume $E_1(x) = E_1 e^{-|x|/a}$, with the voltage $V_1 \equiv -\int_{-\infty}^{\infty} dx E_1(x) = -2E_1 a$. When the range of the field tends to zero, $E_1(x)$ reduces to a δ -function. The spatial dependence of the DC part of the electric field does not need to be specified, as only the voltage $V_0 = -\int_{-\infty}^{\infty} dx E_0(x)$ is of importance in DC transport [9].

The current at the barrier is given by the expectation value $I(x=0, t) = \langle j(x=0, t) \rangle$, where the current operator is defined via the continuity equation, $\partial_x j(x, t) = -e \partial_t \rho(x, t)$. For a high barrier, the tunneling contribution to the current can be expressed in terms of forward and backward scattering rates which are proportional to the tunneling probability Δ^2 . The latter may be obtained in terms of the barrier height U_t by using the instanton approximation [24]. The result can be written in terms of the one-electron propagator $S + iR$ [10],

$$I(x=0, t) = e \Delta^2 \int_0^{\infty} d\tau e^{-S(\tau)} \sin R(\tau) \sin \left[\frac{e}{\hbar} \int_{t-\tau}^t dt' V_{\text{eff}}(t') \right], \quad (5)$$

with

$$S(\tau) + iR(\tau) = \frac{e^2}{\pi \hbar} \int_0^{\omega_{\text{max}}} \frac{d\omega}{\omega} \mathcal{R}e \{ \sigma^{-1}(x=0, \omega) \} \left[(1 - \cos \omega \tau) \coth \frac{\beta \omega}{2} + i \sin \omega \tau \right], \quad (6)$$

where $\beta = 1/k_B T$, ω_{max} the usual frequency cutoff that corresponds roughly to the Fermi energy [23], and the AC conductivity of the system without impurity is [9]

$$\sigma(x, \omega) = \frac{-i v_F e^2 \omega}{\hbar \pi^2} \int_0^{\infty} dk \frac{\cos kx}{\omega^2(k) - (\omega + i0^+)^2}. \quad (7)$$

Furthermore, the effective driving voltage is related to the electric field by [9]

$$V_{\text{eff}}(t) = \int_{-\infty}^{\infty} dx \int_{-\infty}^t dt' E(x, t') r(x, t - t') = V_0 + \frac{\hbar \Omega}{e} |z| \cos(\Omega t - \varphi_z), \quad (8)$$

where $|z|$ and φ_z are, respectively, modulus and argument of z (cf. eq. (2)). The modulus $|z|$ will be the argument of the Bessel functions in the final result, φ_z will represent a phase shift of the harmonics. With the above assumptions about the shapes of the driving field and the interaction potential one obtains

$$|z| = \frac{e V_1}{\hbar \Omega} \frac{1}{\sqrt{1 + a^2 k^2(\Omega)}} A \left(\frac{\Omega}{v_F \alpha}, \frac{k(\Omega)}{\alpha}, \alpha a \right), \quad (9)$$

where $k(\Omega)$ is the inverse of the dispersion relation and

$$A^2(u, v, w) = \frac{1}{1 + u^2} \left[1 + v^2 \frac{(u + vw)^2}{(uw + v)^2} \right]. \quad (10)$$

In the following, we concentrate on the results for the DC component of the current which does not depend on x and is directly given by the current at the barrier, for which we only need to know only $|z|$,

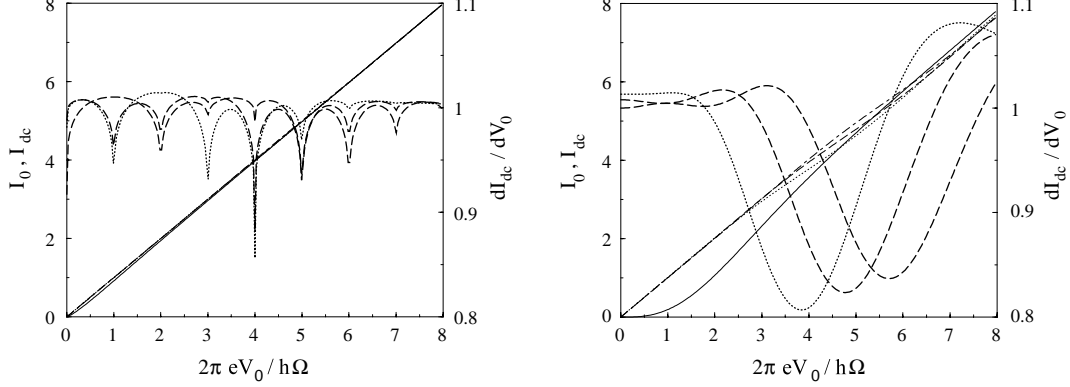


Fig. 1. – Currents I_0 , I_{dc} and differential conductance dI_{dc}/dV_0 at zero temperature as a function of the ratio $eV_0/\hbar\Omega$ for $g = 0.9$ (left), $g = 0.5$ (right) for values $\Omega = v_F\alpha$, $a = 0$, and $eV_1/\hbar v_F\alpha = \ell$ ($\ell = 5$ dotted, $\ell = 6$ dashed, $\ell = 7$ dash-dotted lines). Currents in units of $\hbar v_F\alpha/eR_t$; differential conductance in units of R_t^{-1} ; tunneling resistance $R_t = 2\hbar\omega_{\max}^2/\pi e^2\Delta^2$.

$$\begin{aligned}
 I_{dc} &= e\Delta^2 \int_0^\infty d\tau e^{-S(\tau)} \sin R(\tau) \sin\left(\frac{eV_0\tau}{\hbar}\right) J_0\left(2|z|\sin\frac{\Omega\tau}{2}\right) \\
 &= \sum_{n=-\infty}^{\infty} J_n^2(|z|) I_0\left(V_0 + n\frac{\hbar\Omega}{e}\right). \tag{11}
 \end{aligned}$$

The important point here is that the driven DC current is completely given in terms of $I_0(V_0)$, the nonlinear DC current-voltage characteristic of the tunnel barrier,

$$I_0(V_0) = e\Delta^2 \int_0^\infty d\tau e^{-S(\tau)} \sin R(\tau) \sin\left(\frac{eV_0\tau}{\hbar}\right). \tag{12}$$

Equations (11), (12) generalise results which have been obtained earlier [1] but *without* interaction between the tunneling objects, and also for the Luttinger model with a zero-range interaction, together with a δ -function like driving electric field [8].

For V_0 much smaller than some cutoff voltage V_c which is related to the inverse of the interaction range, $I_0 \propto V_0^{2/g-1}$. This recovers the result obtained earlier for δ -function interaction and zero-range bias electric field [7]. When $V_0 \gg V_c$, the current becomes linear [18]. For intermediate values of V_0 , I_0 exhibits a cross-over between the asymptotic regimes with a point of inflection near V_c . For zero-range interaction, $I_0 \propto V_0^{2/g-1}$ for any V_0 . For the understanding of the driven current-voltage characteristic to be discussed below, this general shape of $I_0(V_0)$ for a finite-range interaction will turn out to be very important.

Figure 1 shows the currents I_0 , I_{dc} and the differential conductance dI_{dc}/dV_0 as functions of $eV_0/\hbar\Omega$ for $g = 0.9$ and $g = 0.5$ for zero-range of the driving electric field. For $g = 0.9$ one observes sharp minima in the differential conductance at integer multiples of the driving frequency in certain regions of the driving voltage V_1 . These can be understood by the following argument. When the strength of the interaction is not too large, the region where dI_{dc}/dV_0 is much smaller than 1 is small compared with $\hbar\Omega$ such that for $eV_0 \approx \hbar\Omega$, $dI_{dc}/dV_0 \propto (2/g-1)|eV_0 - \hbar\Omega|^{2/g-2}$. Then, eq. (11) yields near $eV_0 = m\hbar\Omega$,

$$\frac{dI_{dc}}{dV} \approx 1 - J_m^2(|z|) + \text{const} \cdot J_m^2(|z|) |eV_0 - m\hbar\Omega|^{2/g-2}. \tag{13}$$

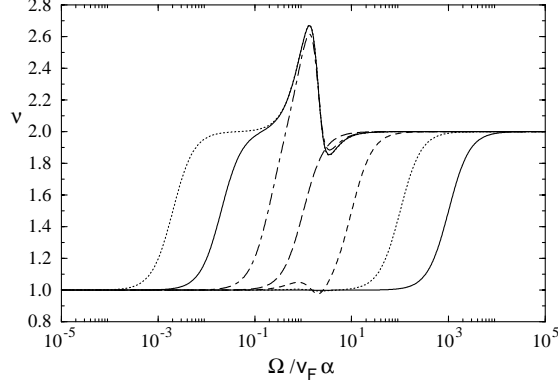


Fig. 2. – Scaling exponent ν of the argument $|z|$ of the Bessel functions as a function of Ω , for ranges of the driving field (curves from right to left) $\alpha a = 10^{-3}, 10^{-2}, 10^{-1}, 1, 10, 10^2, 10^3$, and $g = 0.5$.

For $g > 2/3$, this yields for integer m the cusp-like structures observed in fig. 1. For $g < 2/3$, no cusps occur anymore. In addition, the current I_{dc} is depleted so strongly and over such a large region of the bias voltages that the regime of almost vanishing dI_{dc}/dV_0 becomes larger than $\hbar\Omega$ and in general no minima near integer multiples of the frequency exist. As can be seen in the figure, the depths of the cusps depend on the driving voltage V_1 ($\propto |z|$) which can also be understood from of eq. (13) which shows that the values of the differential conductances at the voltages $eV_0 = m\hbar\Omega$ are approximately $1 - J_m^2(|z|)$.

It is therefore instructive to look into the behaviour of $|z|$ as a function of the frequency. Figure 2 shows the scaling exponent ν determined from

$$\nu = -v_F\alpha \frac{d \log |z|}{d \log \Omega}. \quad (14)$$

We observe a non-universal cross-over between $|z| \propto \Omega^{-1}$, the case discussed by Tien and Gordon [1] which corresponds to a driving field of zero-range ($a \rightarrow 0$), and $|z| \propto \Omega^{-2}$ which is obtained for a homogeneous external field ($a \rightarrow \infty$) [3]. Although the behaviour of z depends strongly on the parameters of the model in the cross-over regime, this does not influence qualitatively the occurrence of the cusps. Their existence depends crucially on the finite range of the interaction, and the condition $g > 2/3$. However, by varying $|z|$, the depths of the minima are changed due to the variation of $J_m^2(|z|)$.

Given the above result for the driven DC current, the general behaviour of the differential conductance as a function of $eV_0/\hbar\Omega$ can be straightforwardly obtained. Of special interest is the occurrence of cusps at $eV_0/\hbar\Omega = m$ (m integer) which appear to be quite stable against changes in the model parameters. A similar result has been discussed earlier [25], but for a small potential barrier between fractional quantum Hall edge states which implies zero-range interaction. In the general case discussed here, the finite range of the interaction is crucial for obtaining the cusps, due to the absence of a linear contribution towards the current for small voltage which is characteristic of tunneling in 1D dominated by interaction. The cusps could be used to frequency-lock the DC part of the driving voltage.

In conclusion, we have demonstrated that the result which has been obtained by Tien and Gordon for tunneling of non-interacting quantum objects in 1D driven by a monochromatic

field localised at the tunnel barrier remains valid even in the presence of interactions of arbitrary range and shape, and for an arbitrary shape of the monochromatic driving field. The central point is that the frequency-driven current is completely given by a linear superposition of the current-voltage characteristics at integer multiples of the driving frequency, weighted by Bessel functions.

The argument of the latter contains the amplitude of the driving voltage only linearly but the dependence of the argument on the frequency and the range of the driving field is determined by its spatial shape. However, one can easily identify regions where the dependence on the frequency becomes very simple. For a driving field which is localised near the tunnel barrier, the integral in eq. (2) can be evaluated approximately by noting that $r(x, \Omega)$ varies only slowly with x and can be taken out of the integral. Then, $|z| = eV_1/\hbar\Omega$ which corresponds to the result of Tien and Gordon [1]. In the other limit of an almost homogeneous electric field, $E_1 = V_1/a$, one needs to calculate the spatial average of $r(x, \Omega)$ [9]. This gives $\sigma(k=0, \Omega)/\sigma(x=0, \Omega) \approx \Omega^{-1}$, since $\sigma(x=0, \Omega) \approx \text{const.}$ This implies $|z| \propto \Omega^{-2}$. Such a frequency dependence has been discussed earlier for non-interacting particles [3]. Here, we see that it is valid under quite general assumptions also for interacting particles. A possible method to detect this behaviour experimentally is to investigate the real part of the first harmonic of the current through the tunnel contact and to determine the *current responsivity* which is given by the ratio of the expansions of I_{dc} and the first harmonic to second and first order in $|z|$, respectively [26].

Finally, we wish to comment on the question of self-consistency [11]. The driving field has been assumed *ad hoc* in the present paper, and not determined self-consistently by taking into account the internal interaction-induced rearrangement of charges and currents as a consequence of an external field. There are basically two effects that have to be considered.

First, rearrangement of the charges and currents will lead to a change of the spatial shape of the driving field. As discussed above, the present results allow to identify features that are independent of the spatial shape of the field and thus can be expected to survive even if self-consistent corrections are taken into account. Second, apart from the displacement current contribution, which is present even for $\Delta \rightarrow 0$ and renormalises $|z|$, additional contributions to the local field with other frequencies may be generated, especially in the present nonlinear case. Even the DC current could be changed due to mixing of harmonics present in the local field. If self-consistency maps contributions with different frequencies into the local field, possibly the above result have to be generalised. Strictly speaking, the Tien-Gordon form will no longer be valid. However, as has been discussed earlier [10], the amplitudes of the n -th harmonics generated by the tunnel barrier when starting from a monochromatic external voltage decay extremely rapidly with n . Thus, when using them as a starting point for local field corrections, practically only the fundamental frequency would contribute. This would eventually lead only to changes in the spatial behaviour of E_1 and not destroy the existence of the cusps. In summary, we claim that local field corrections which need to be treated in any case beyond the mean-field approximation are in the present context unimportant and would not lead to essential changes of our findings. This concerns especially the existence of the cusps in the differential conductance at very low temperatures that are characteristic of the non-linearity induced by the finite-range interaction. But further, more detailed studies are necessary, in order to clarify the quantitative aspects of this issue [27].

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