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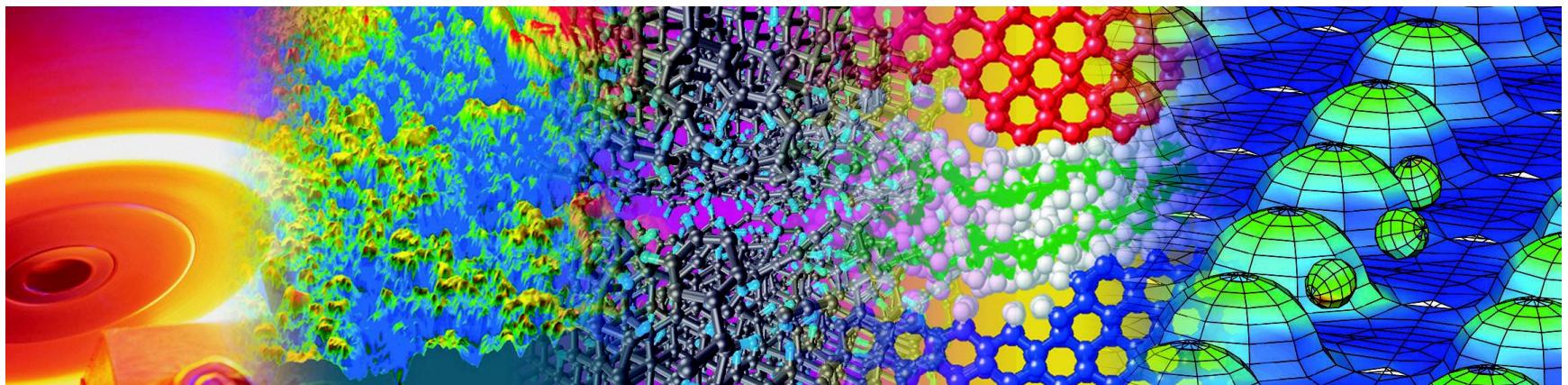
# ***ATOMISTIC/CONTINUUM MULTISCALE COUPLING***

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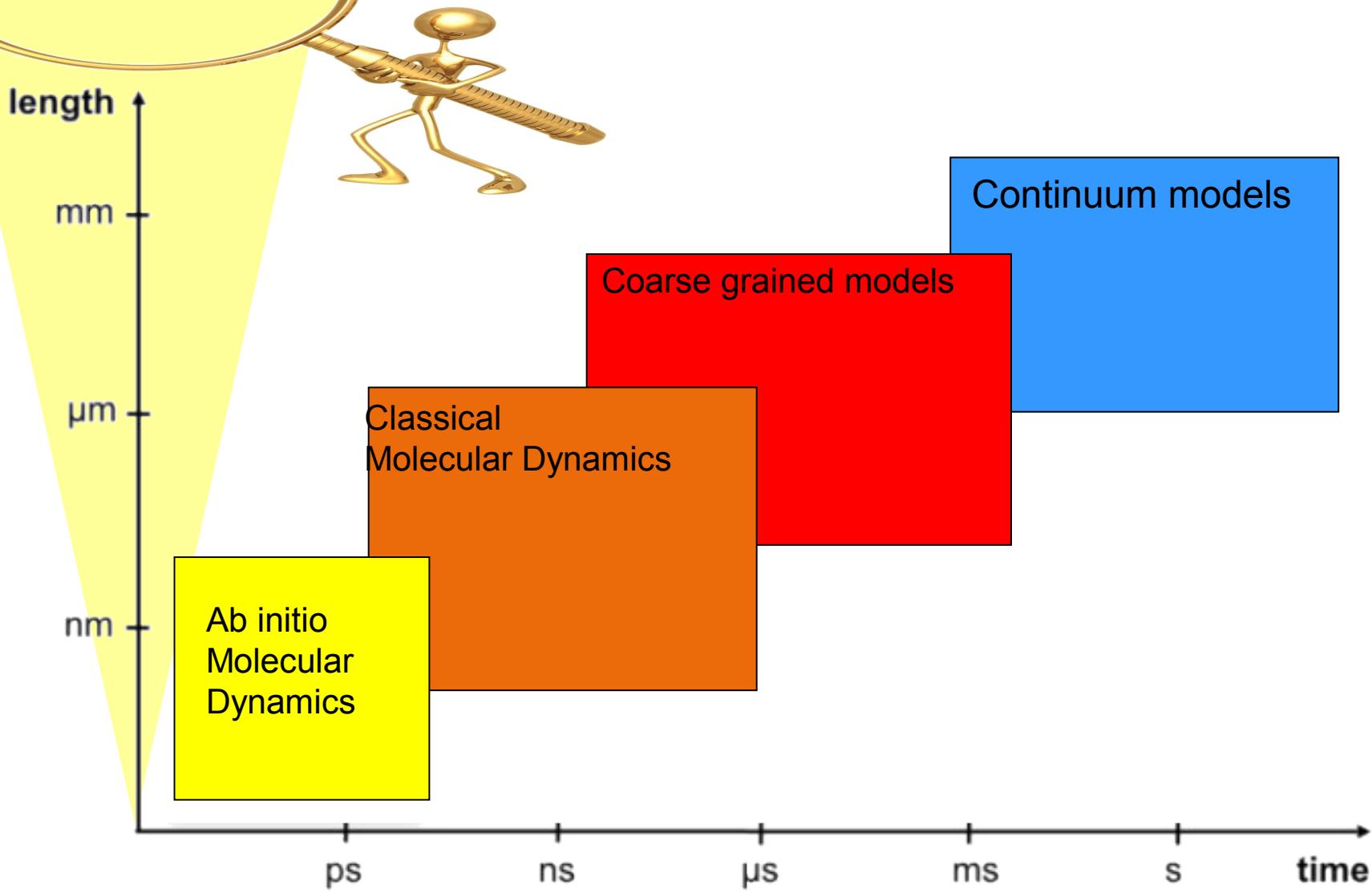
Michael Moseler

Multiscale Modelling and Tribosimulation

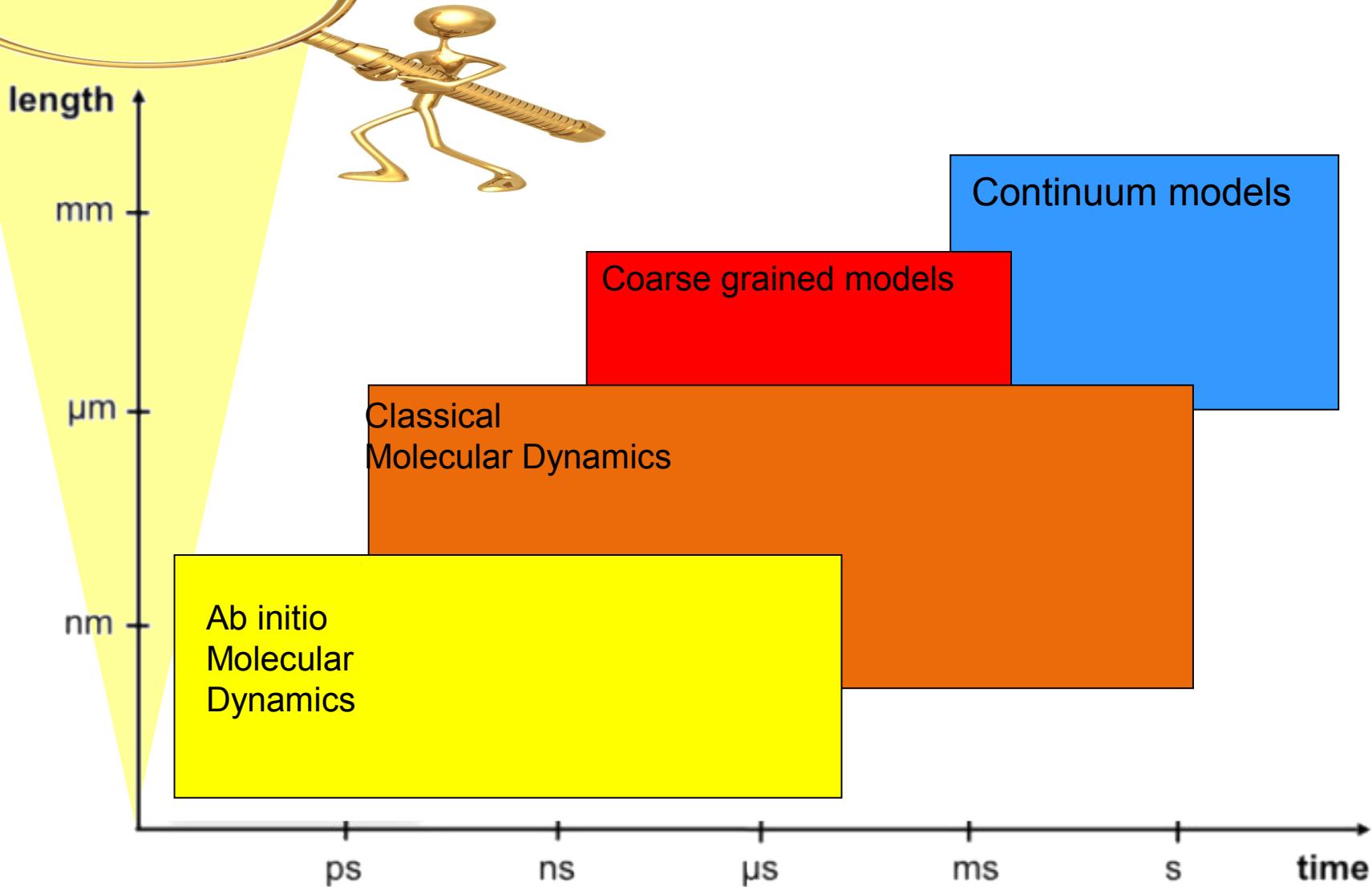
Fraunhofer Institute for Mechanics of Materials IWM



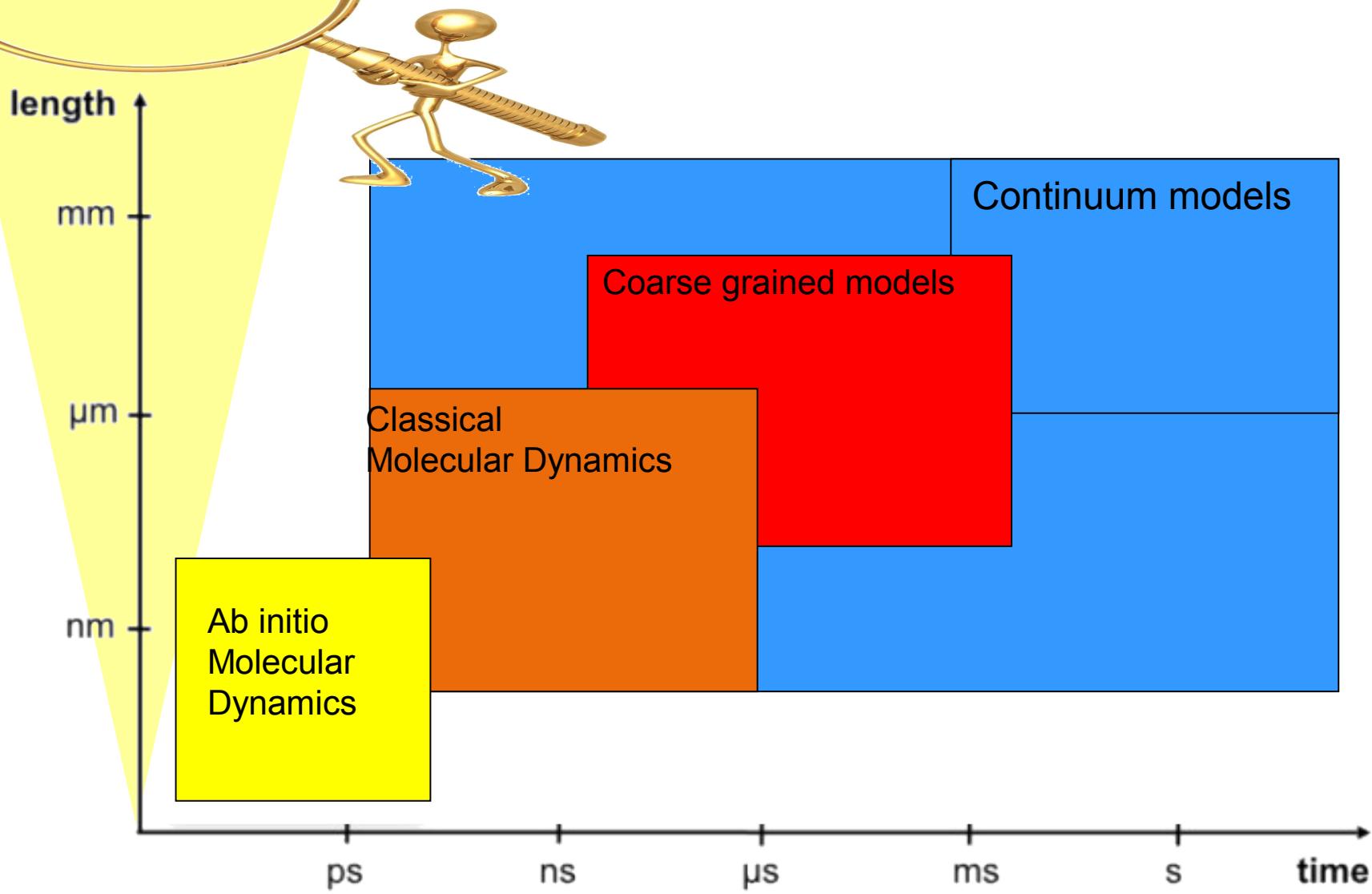
# Multiscale Materials Modelling (MMM)



# Accelerated Molecular Dynamics

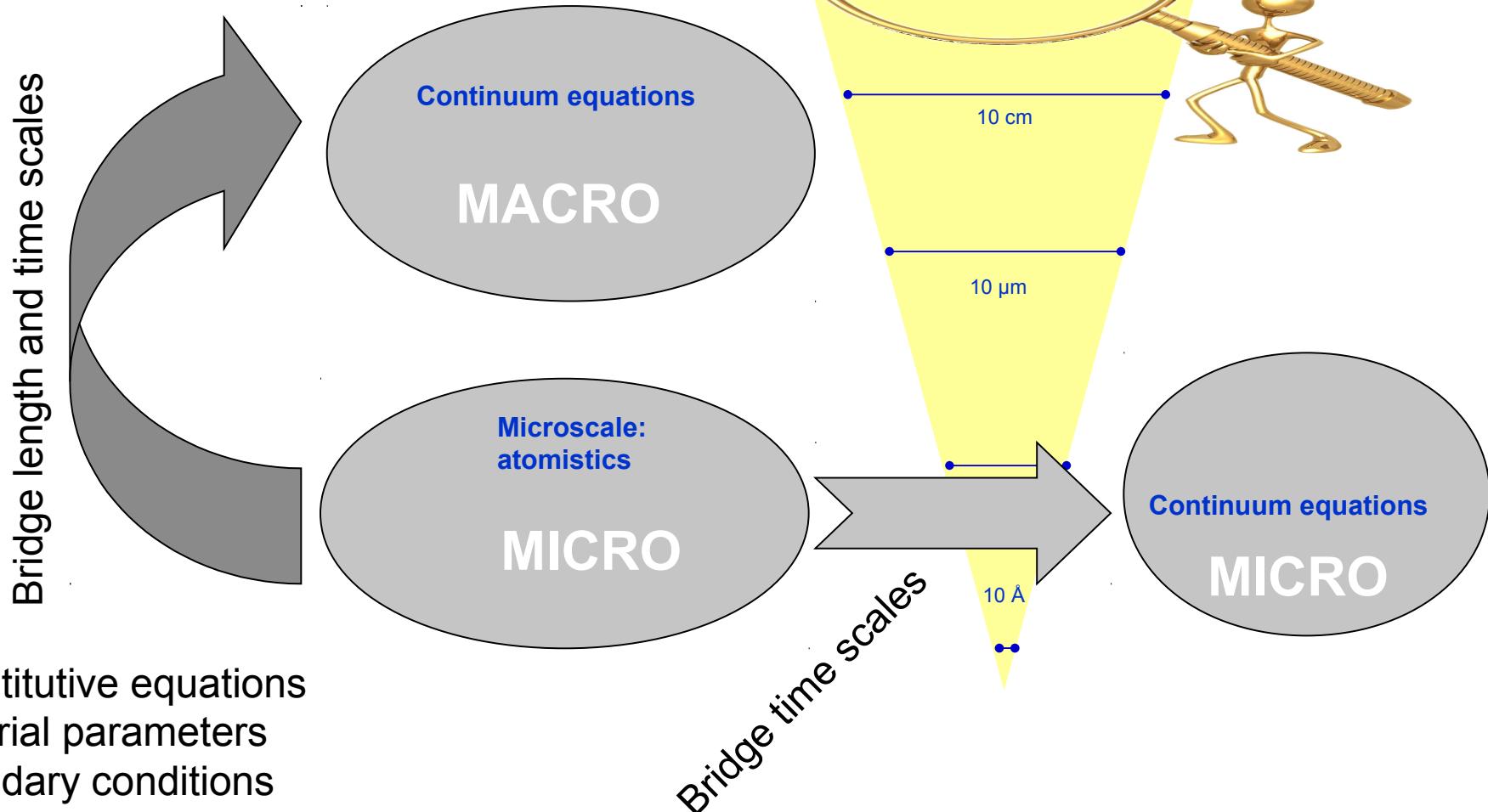


# Extending the continuum regime



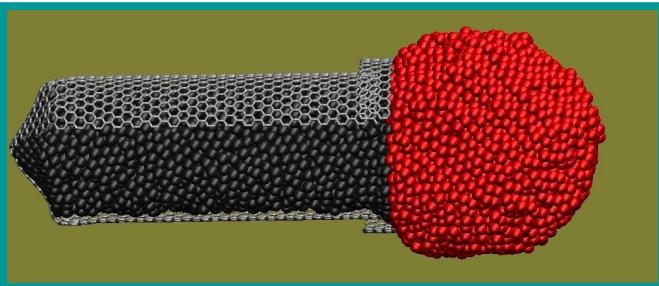
# Multiscale modeling

## Hierarchical atomistic/continuum coupling

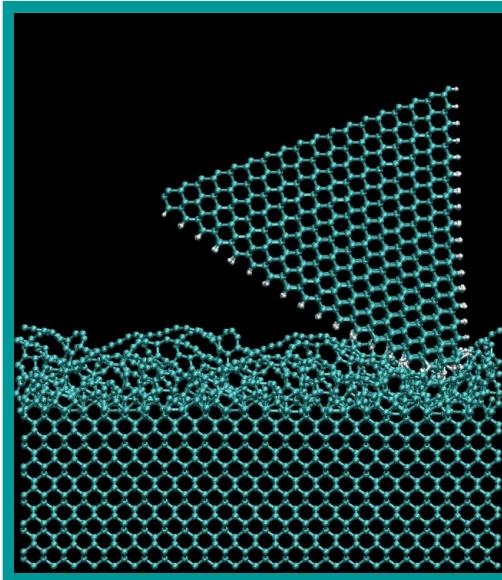


# *Classical transport and shape evolution in small systems*

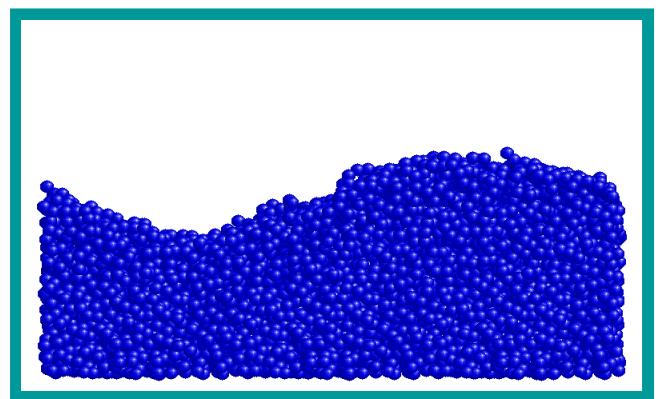
Today



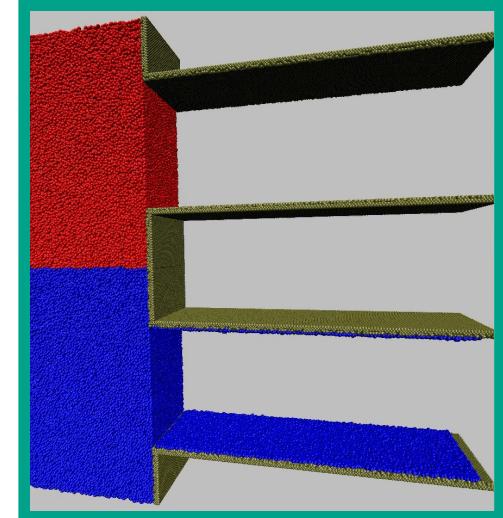
CNT growth:  
ACS Nano 5, 686 (2011)



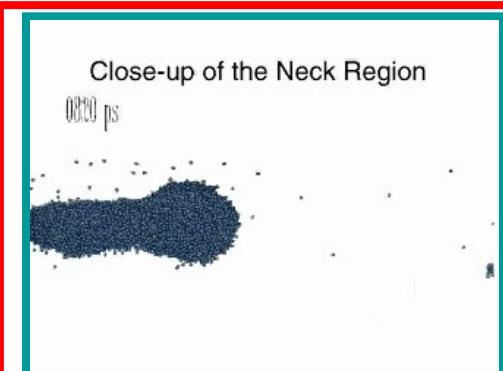
Wear of surfaces:  
Nature Mat. 10, 34 (2011)



DLC growth:  
Science 309, 1545 (2005)

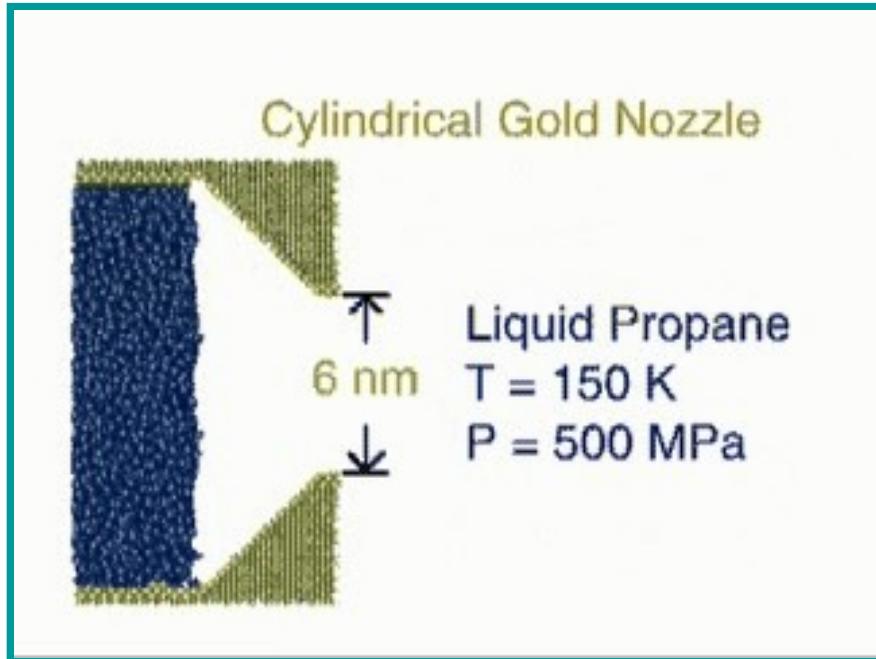
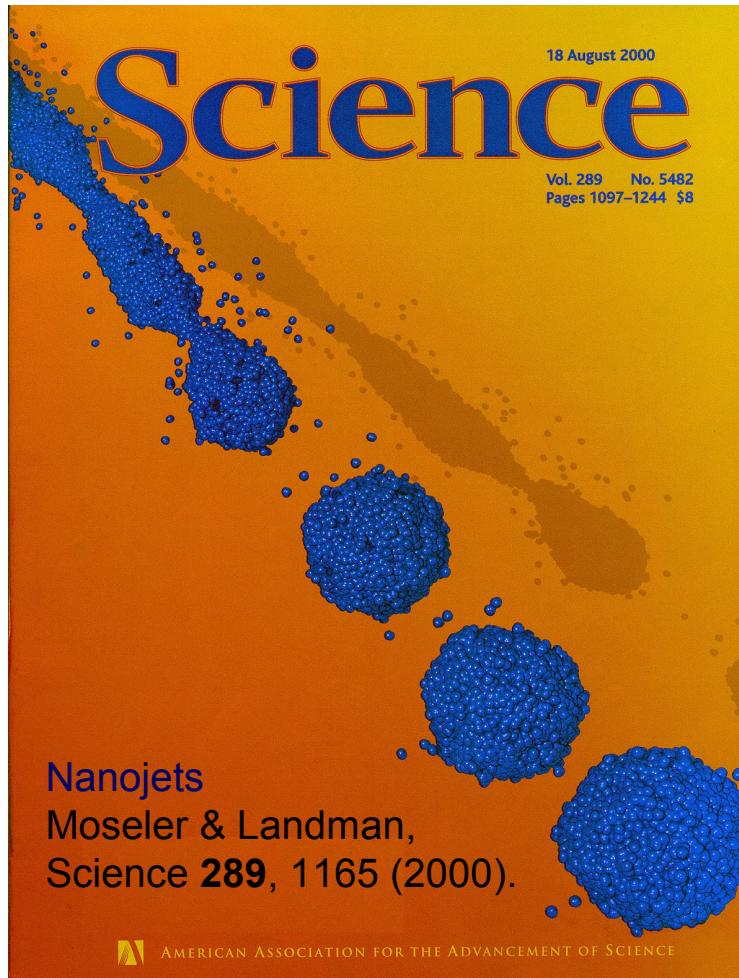


Capillary impregnation:  
NJP, 10, 113022 (2008)



Rayleigh instability:  
Science 289, 1165 (2000)

# Classical molecular dynamics of nanojet formation and breakup



# Rayleigh instability and the validity of hydrodynamics on the nanoscale?

Lubrication equations:

$$\partial_t v + v \partial_z v = -\frac{\gamma \partial_z \kappa}{\rho} + 3\nu \frac{\partial_z(h^2 \partial_z v)}{h^2}$$

$$\partial_t h + v \partial_z h = -\frac{h \partial_z v(z, t)}{2}$$

Jens Eggers, Rev. Mod. Phys. **69**, 865,(1997)

J. Eggers, E. Villermaux, Rep. Prog. Phys. **71** , 036601  
(2008)

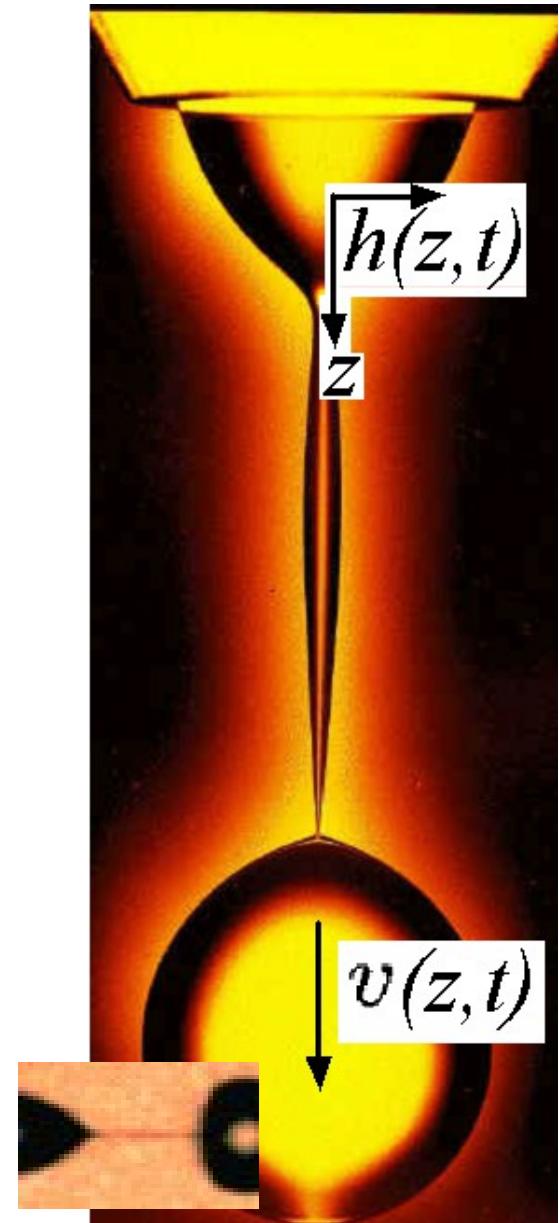
Intrinsic scales:

$$\ell_\nu = \nu^2 \frac{\rho}{\gamma} \quad t_\nu = \nu^3 \left( \frac{\rho}{\gamma} \right)^2$$

$$\partial_t v + v \partial_z v = -(\partial_z \kappa) + 3\partial_z(h^2 \partial_z v)/h^2$$

Similarity solution :  
→ threads

$$h_{\text{neck}} \ll \ell_\nu$$



Navier-Stokes:

$$\rho(\partial_t v_i(\mathbf{r}, t) + v_j(\mathbf{r}, t)\partial_j v_i(\mathbf{r}, t)) = \partial_j \sigma_{ji}(\mathbf{r}, t) \quad (1)$$

Stress tensor  $\sigma_{ik}(\mathbf{r}, t) = -p(\mathbf{r}, t)\delta_{ik} +$

$$\eta(\partial_k v_i(\mathbf{r}, t) + \partial_i v_k(\mathbf{r}, t)) + s_{ik}(\mathbf{r}, t)$$

$\partial_t$

Fluctuation-dissipation theorem

$$\partial_t \langle s_{ik}(\mathbf{r}, t)s_{lm}(\mathbf{r}', t') \rangle = 2k_B T \eta (\delta_{il}\delta_{km} + \delta_{im}\delta_{kl} - \frac{2}{3}\delta_{ik}\delta_{lm})\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$$

Expansion

$$\mathbf{v}(r, z, t) = \sum_{i=0}^{\infty} \int_{-\infty}^{\infty} v^{(2i)}(\bar{z}, t) \mathbf{w}^{(2i, \bar{z})}(r, z) d\bar{z}$$

Into divergence-free basis

$$\mathbf{w}^{(2i, \bar{z})}(r, z) = \begin{pmatrix} -\frac{r^{2i+1}}{2i+2} \delta'(z - \bar{z}) \\ r^{2i} \delta(z - \bar{z}) \end{pmatrix}$$

Lubrication approximation

$$\mathbf{v}(\mathbf{r}, t) = \begin{pmatrix} -\frac{x}{2} \partial_z v^{(0)}(z, t) \\ -\frac{y}{2} \partial_z v^{(0)}(z, t) \\ v^{(0)}(z, t) \end{pmatrix}$$

Multiplication of eq. (1) with

$$\mathbf{w}^{(0, \bar{z})}(\mathbf{r})$$

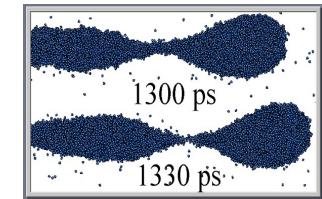
Integration over jet volume

$$V = \{ \mathbf{r} | \sqrt{x^2 + y^2} \leq h(z) \}$$

# Experimental validation?

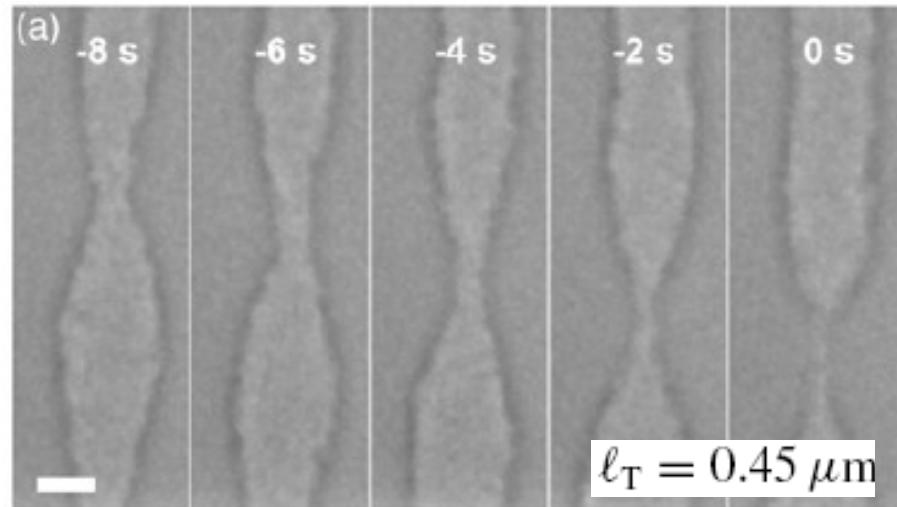
Stochastic terms dominate  
on length scales smaller than  
Thermal capillary length scales:

$$\ell_T = (k_B T / \gamma)^{1/2}$$

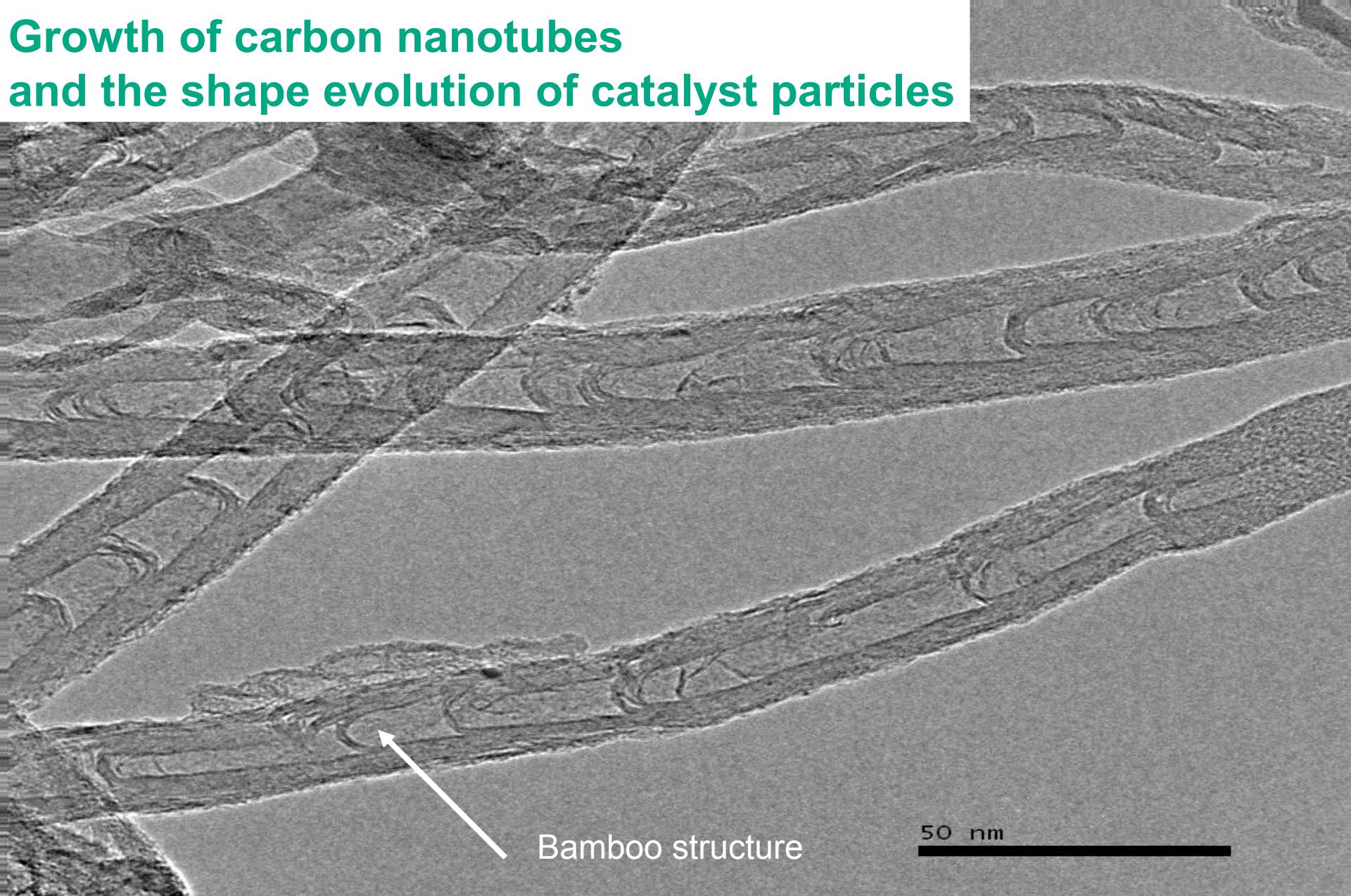


Hennequin, et al., *Phys. Rev. Lett.* **97** 244503 (2006)

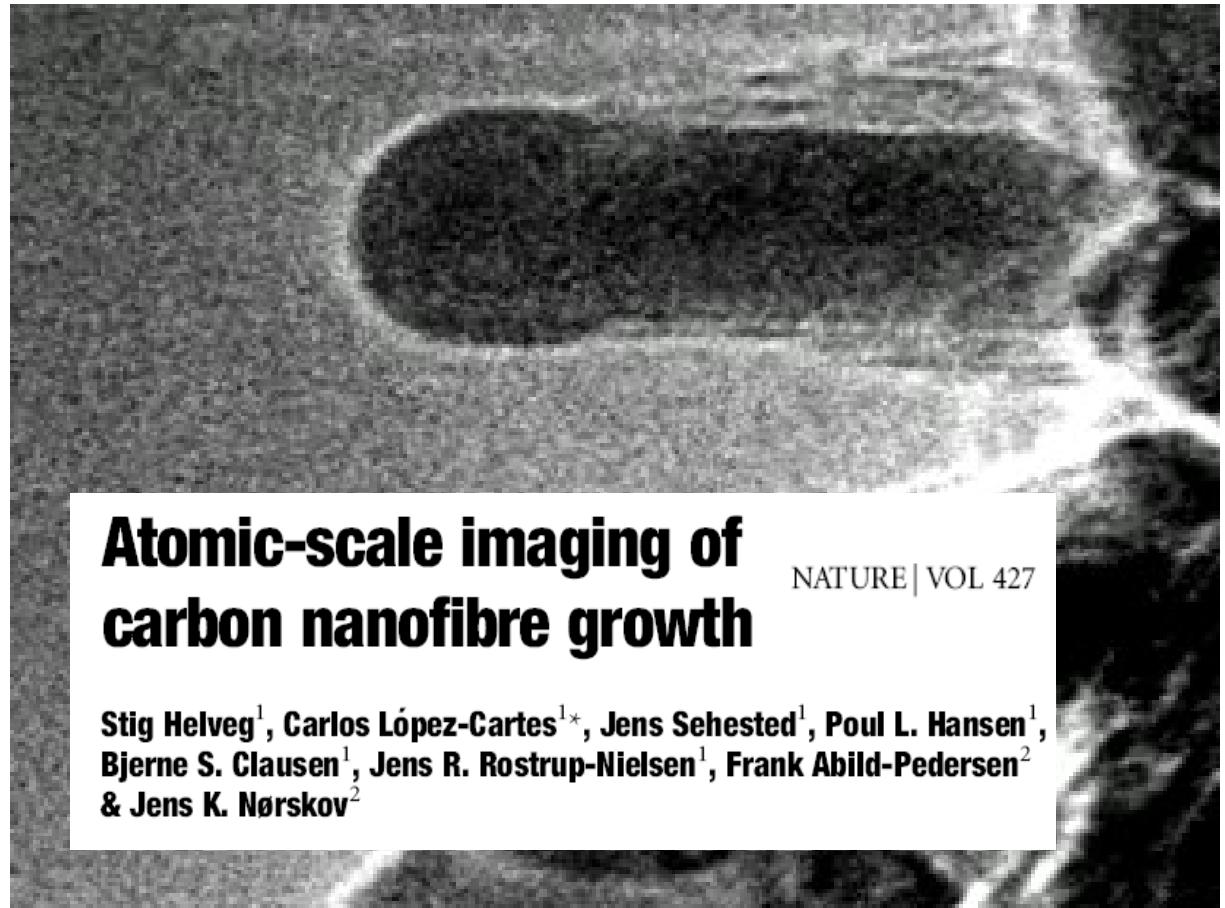
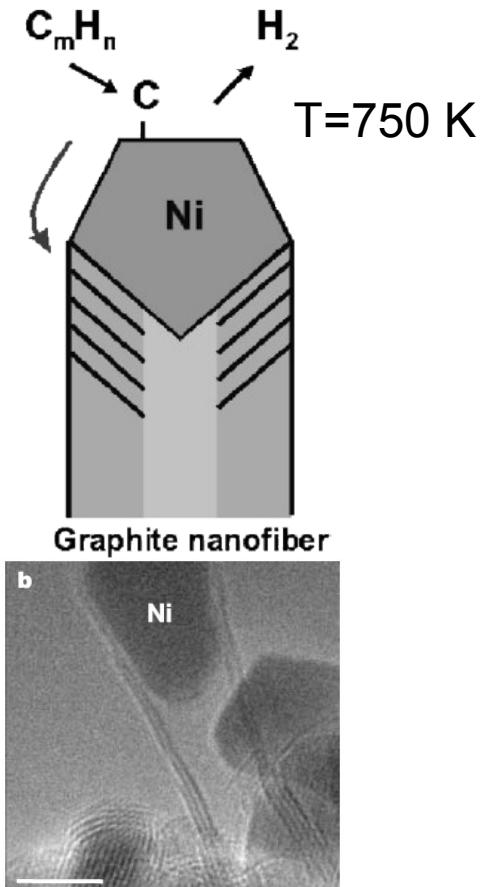
Colloidal fluids:



# Growth of carbon nanotubes and the shape evolution of catalyst particles



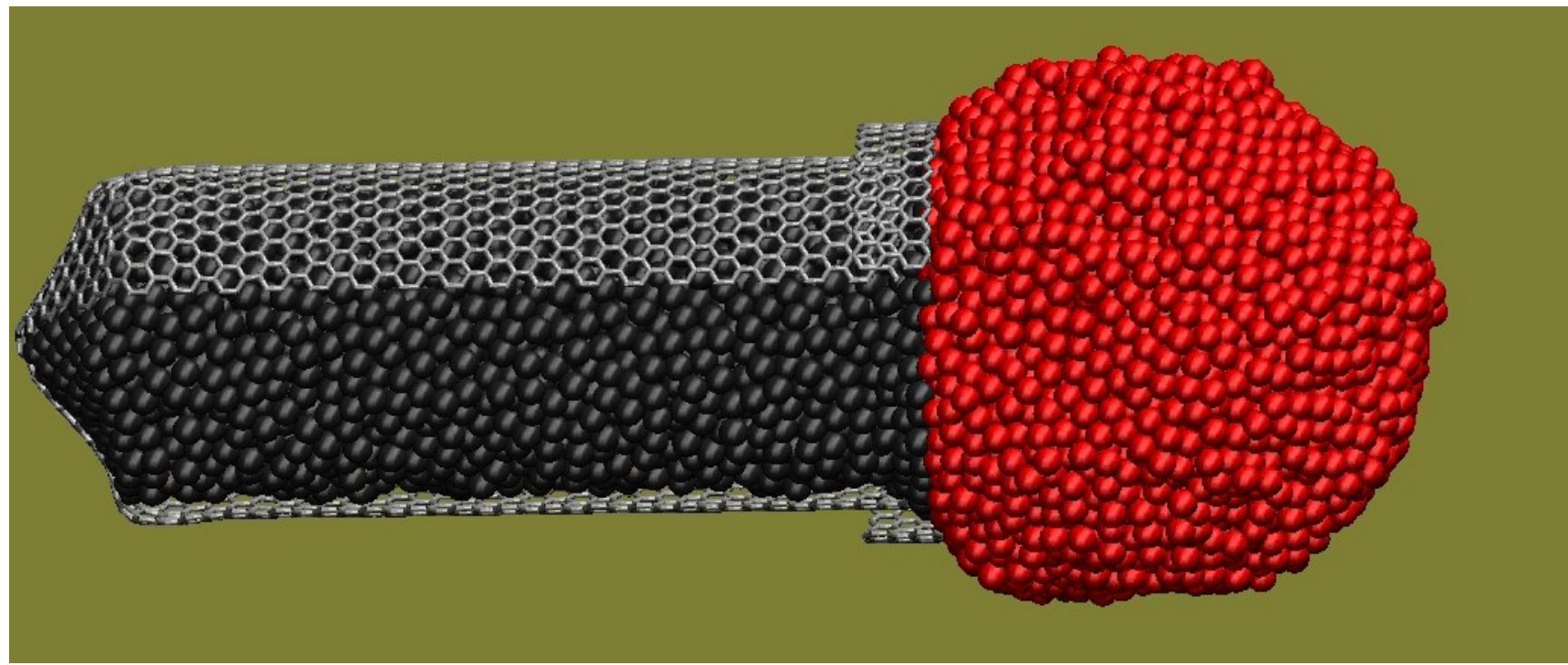
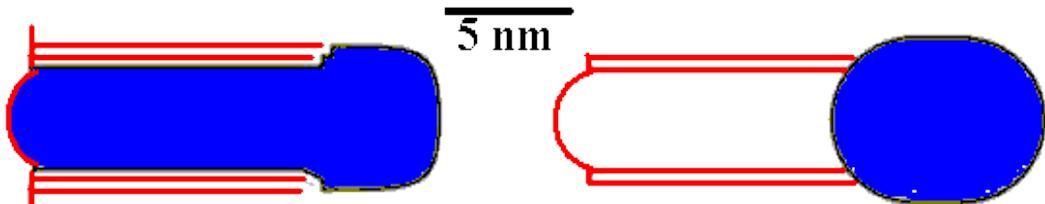
# Environmental TEM experiments: Shape dynamics of Ni catalyst particles



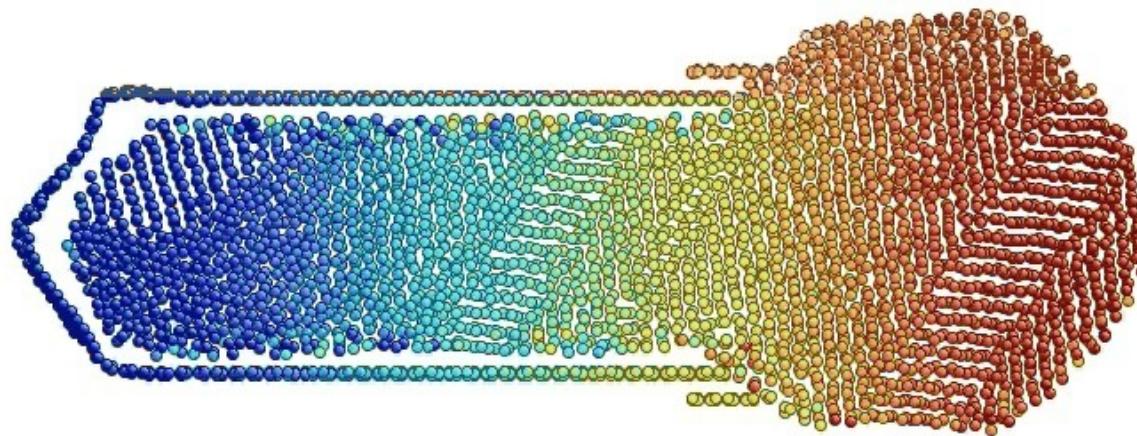
# Molecular dynamics of a solid Ni nanoparticle in a double wall carbon nanotube

Moseler et al.,  
ACS Nano 5, 686 (2011)

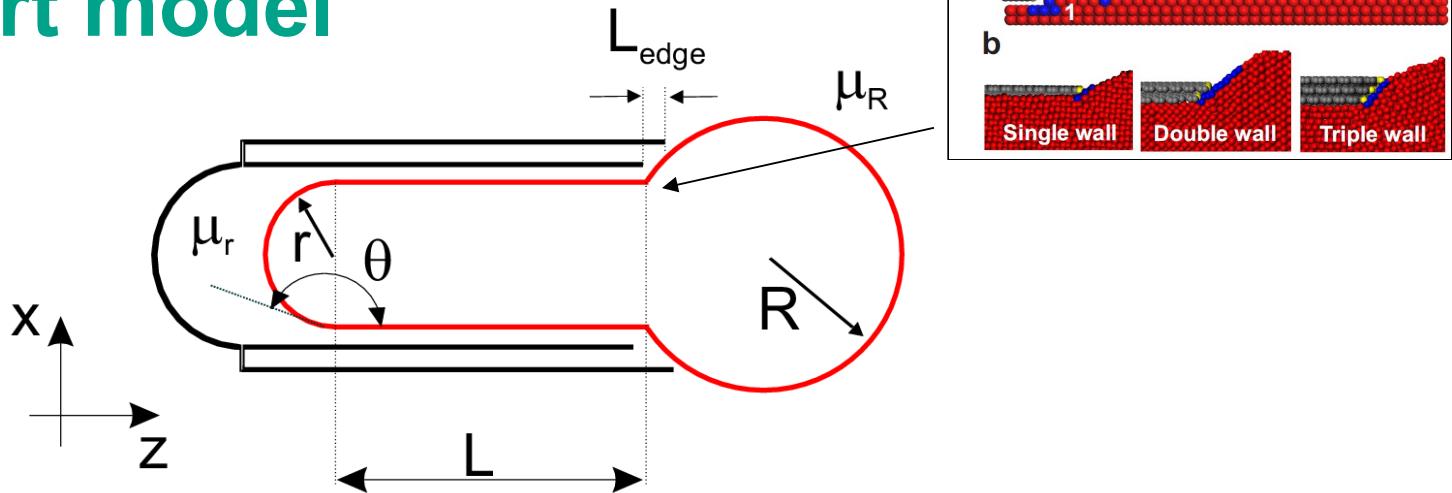
A T=1160 K EAM Ni<sub>10561</sub>  
interacting via Morse potentials  
with a static CNT



# Transport mechanism: surface diffusion



# Continuum transport model



Two Reservoirs:

$$\mu_r = \gamma\Omega \frac{2}{r} \quad \mu_R = \gamma\Omega \frac{2}{R}$$

Diffusive particle current:

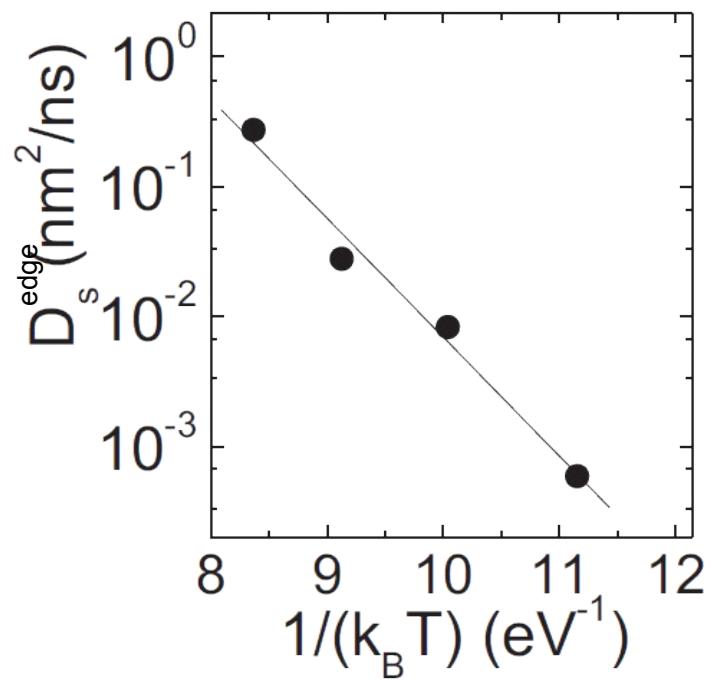
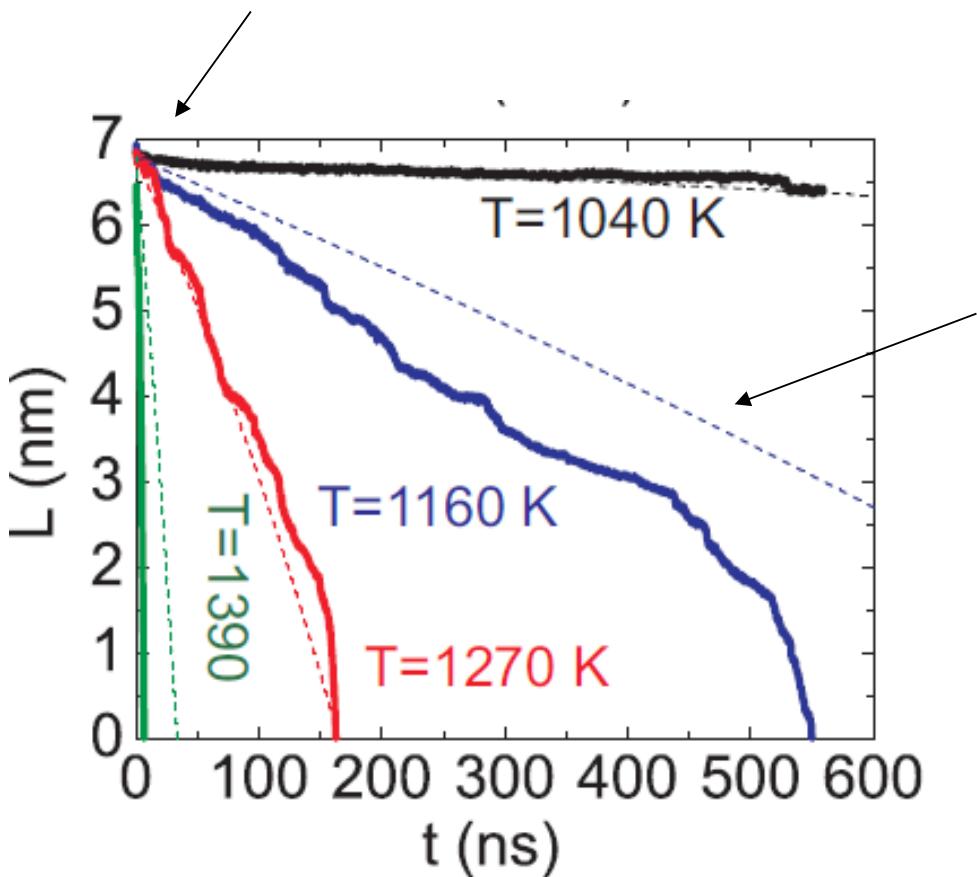
$$J = -2\pi r \frac{D_s^{edge} \rho_s}{kT} \frac{\partial \mu}{\partial z}$$

Mullins B:  
WW Mullins, J.Appl.Phys. 28 333 (1957)

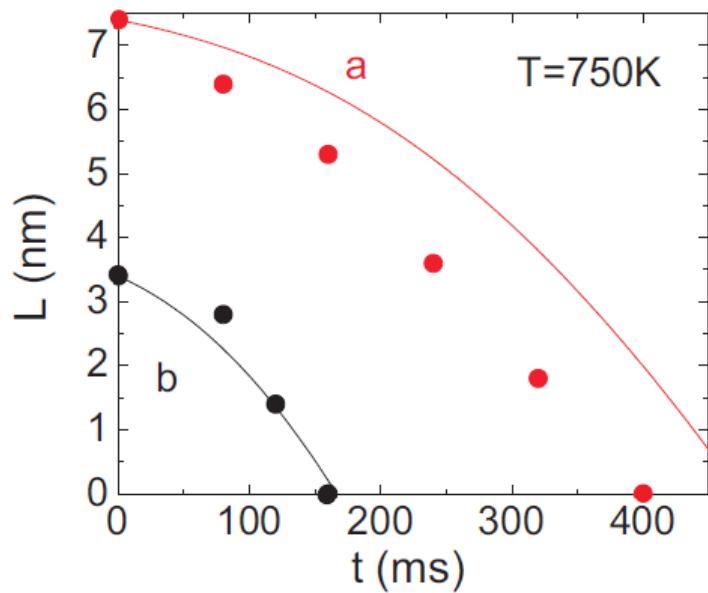
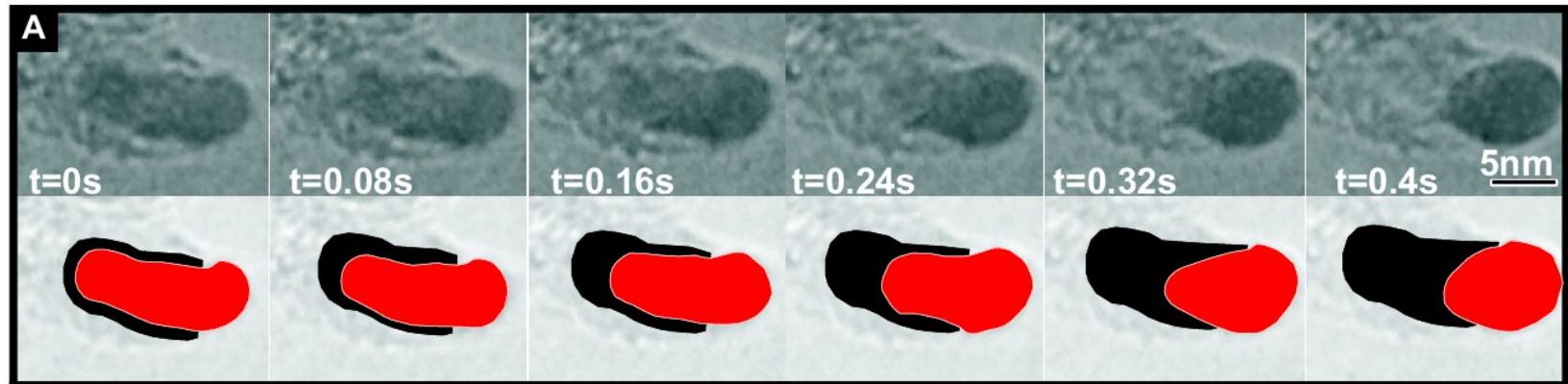
$$\frac{dL}{dt} = -\frac{4B}{L_{edge} r^2} \left[ 1 - \frac{r}{R(L)} \right]$$

$$B = \frac{\gamma D_s^{edge} \rho_s \Omega^2}{kT}$$

Use  $\frac{dL}{dt}\Big|_{t=0}$  from MD and solve  $\frac{dL}{dt} = -\frac{4B}{L_{edge} r^2} \left[ 1 - \frac{r}{R(L)} \right]$  for  $D_s^{\text{edge}}$



# Comparison with experiment

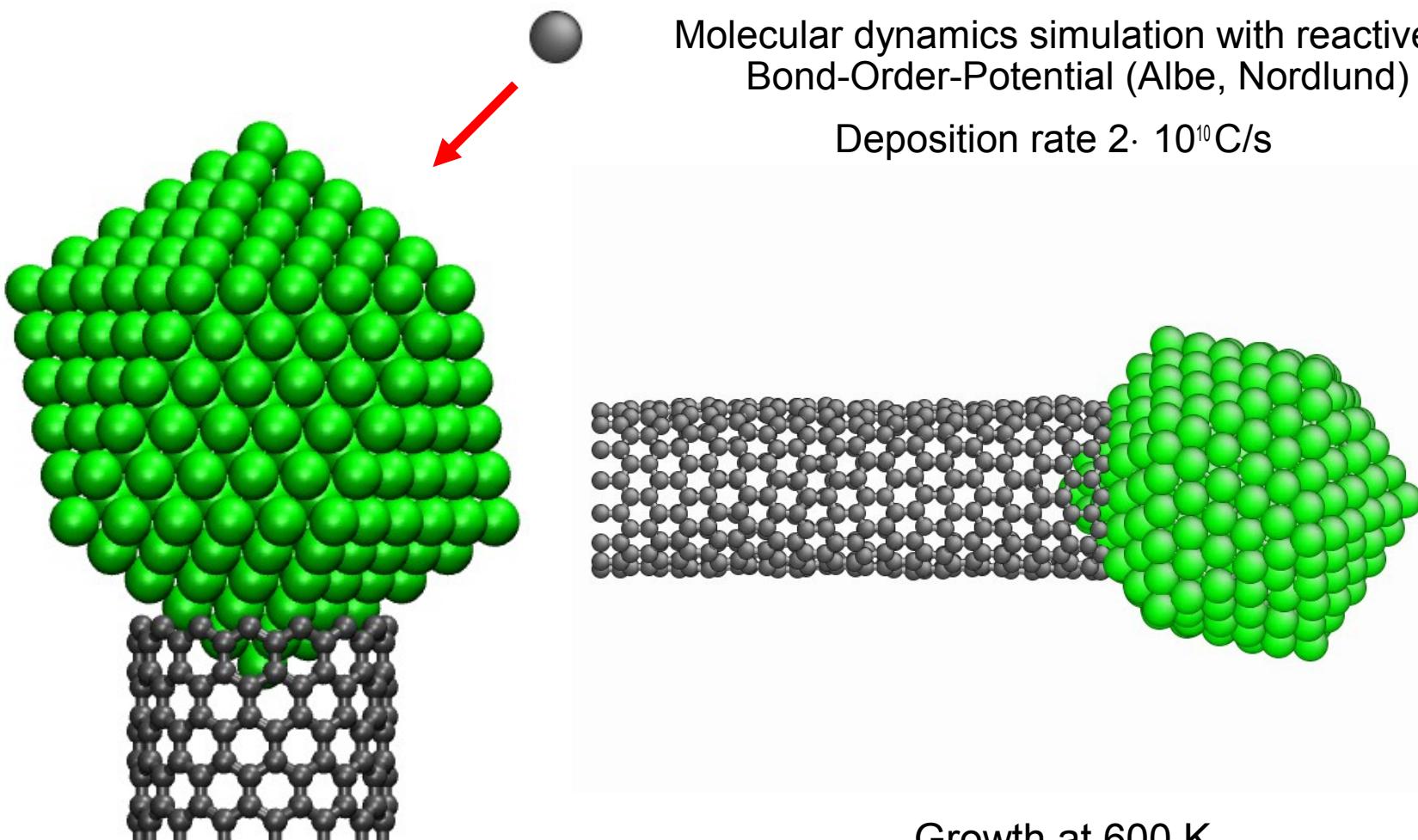


# CNT with Fe-Catalyst

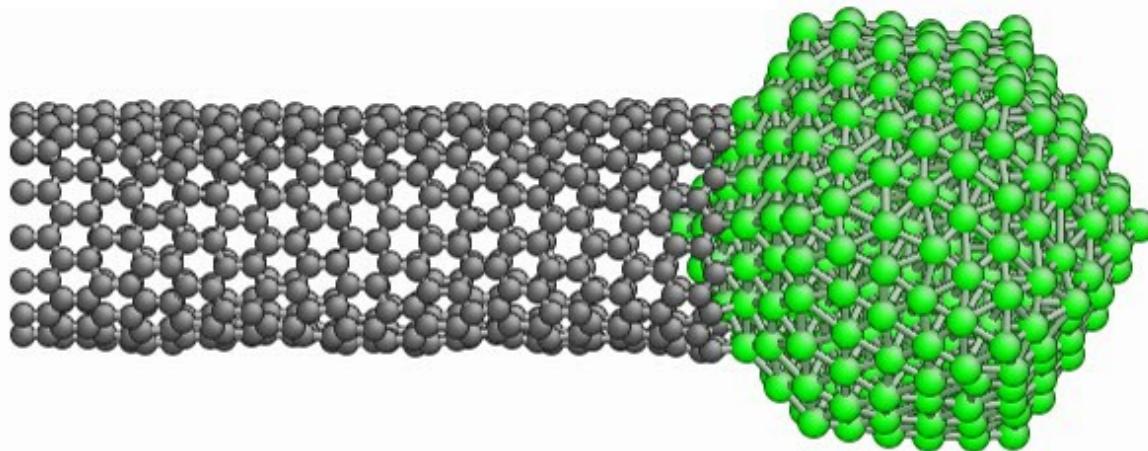
Tip growth of (15,0) tube catalysed by  $\text{Fe}_{561}$

Molecular dynamics simulation with reactive Bond-Order-Potential (Albe, Nordlund)

Deposition rate  $2 \cdot 10^{10} \text{ C/s}$



# Growth at 1200K

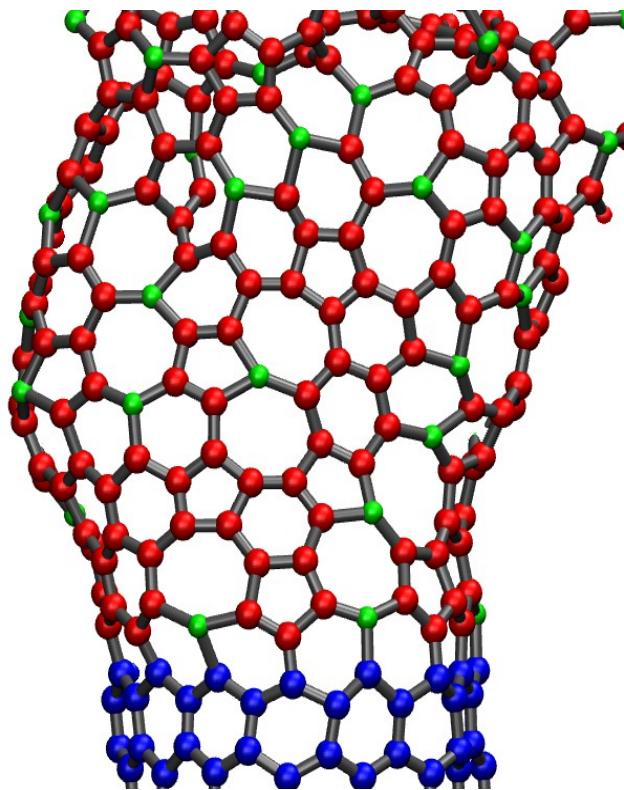


Japanese Journal of Applied Physics  
Vol. 43, No. 4A, 2004, pp. L471–L474  
©2004 The Japan Society of Applied Physics

## ***In Situ* Study of Iron Catalysts for Carbon Nanotube Growth Using X-Ray Diffraction Analysis**

Kenji NISHIMURA<sup>1</sup>, Nobuharu OKAZAKI<sup>2</sup>, Lujun PAN<sup>1,2</sup> and Yoshikazu NAKAYAMA<sup>1,2,\*</sup>

# Cross section of the newly grown CNT



Fe (green) is incorporated in CNT-walls

PRL 102, 126807 (2009)

PHYSICAL REVIEW LETTERS

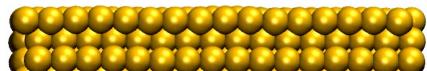
week ending  
27 MARCH 2009

## Embedding Transition-Metal Atoms in Graphene: Structure, Bonding, and Magnetism

A. V. Krasheninnikov,<sup>1,2,\*</sup> P. O. Lehtinen,<sup>1</sup> A. S. Foster,<sup>1,3</sup> P. Pyykkö,<sup>4</sup> and R. M. Nieminen<sup>1</sup>

# Deposition of Ag<sub>N</sub> on Au(111)

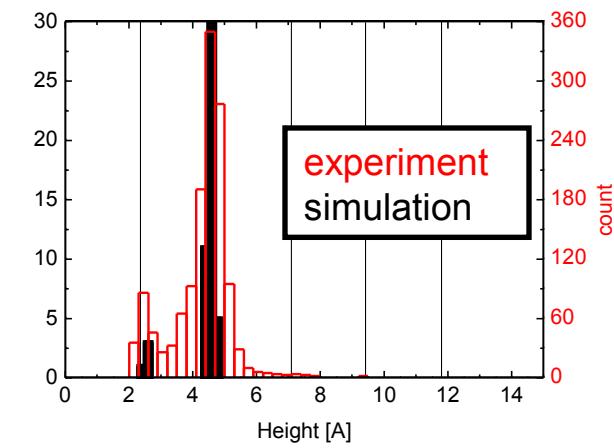
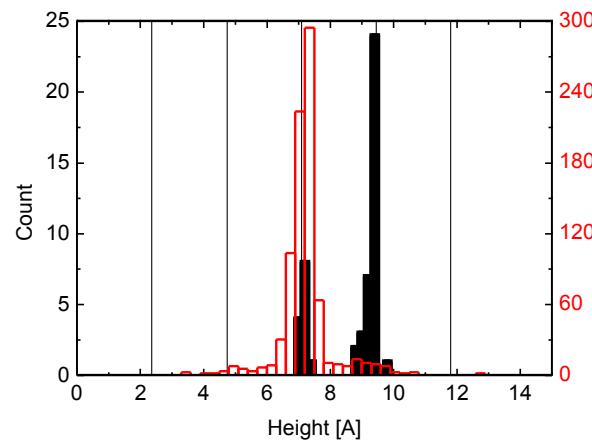
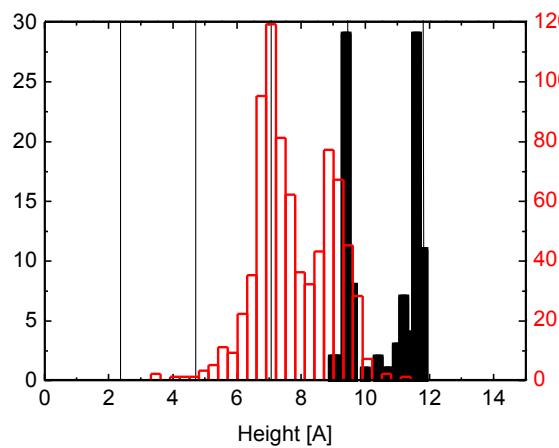
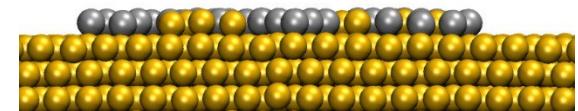
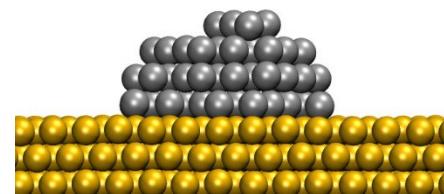
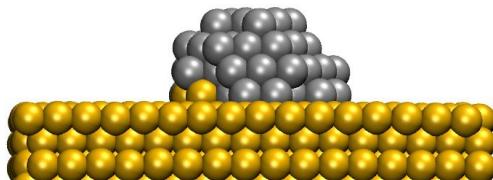
## Effect of deposition energy



3 eV

34 eV

340 eV

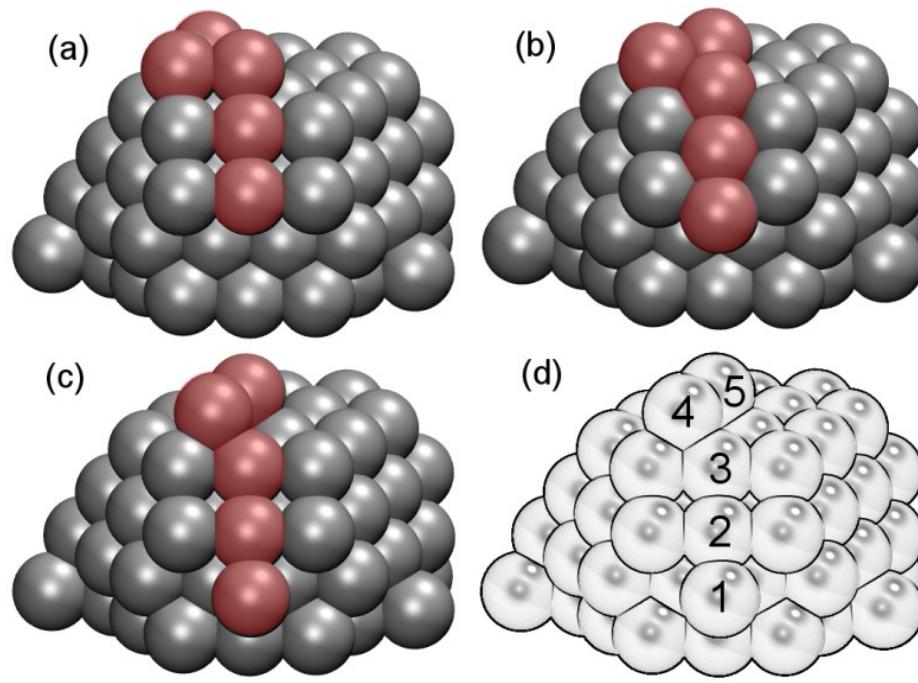
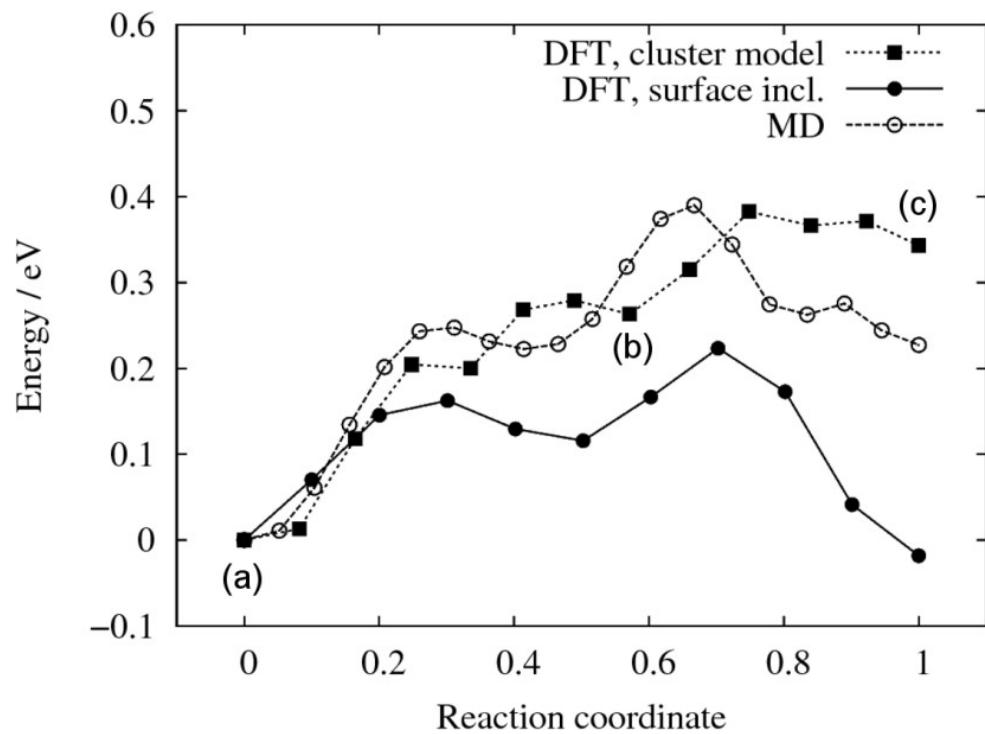
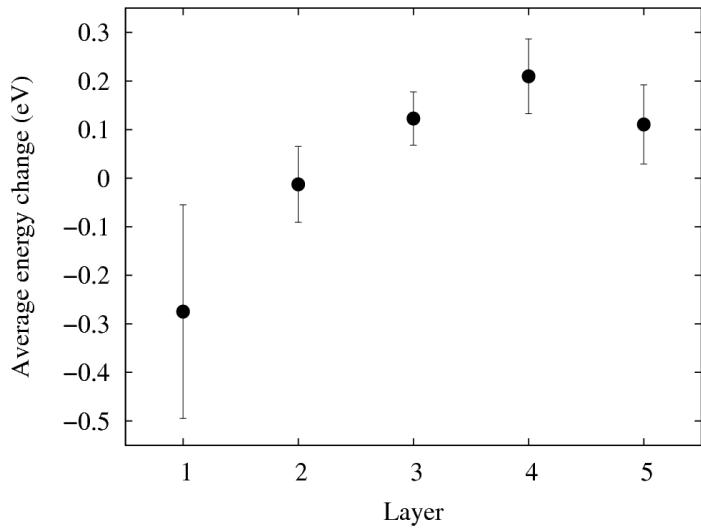


Grönhagen, Järvi et al., to be published

© Fraunhofer-Institut für Werkstoffmechanik IWM

# Decay mechanism

- Decay of top monolayer
  - Non-local effect of surface on cluster  
→ Barrier from ca. 0.4 to 0.2 eV
  - Barrier inferred from experiment at 77 K: ca. 0.25 eV



# Summary

A hierarchical atomistic/continuum modelling is quite usefull for a quantitative understanding of complex shape dynamics in nanoscale systems.

## Acknowledgement:

Uzi Landman, Georgia Institute of Technology

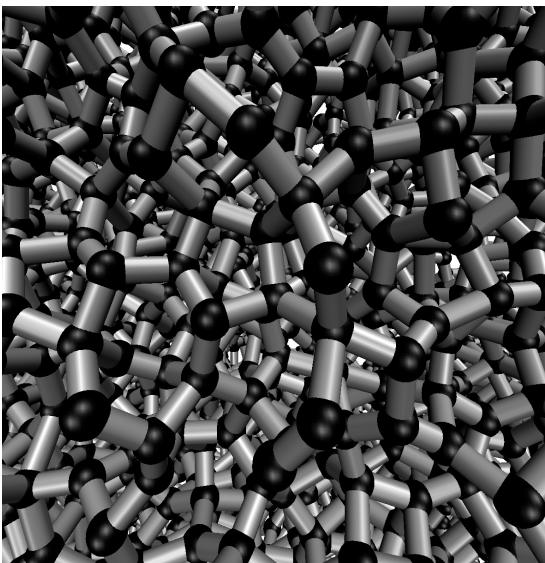
Andreas Klemenz, Fraunhofer IWM

F. Cervantes-Sodi, S.Hofmann, G. Czanyi, A. Ferrari, Cambridge Univ.

Tommi Järvi, Fraunhofer IWM

**Thank you for your attention!**

# Topography evolution during thin film growth



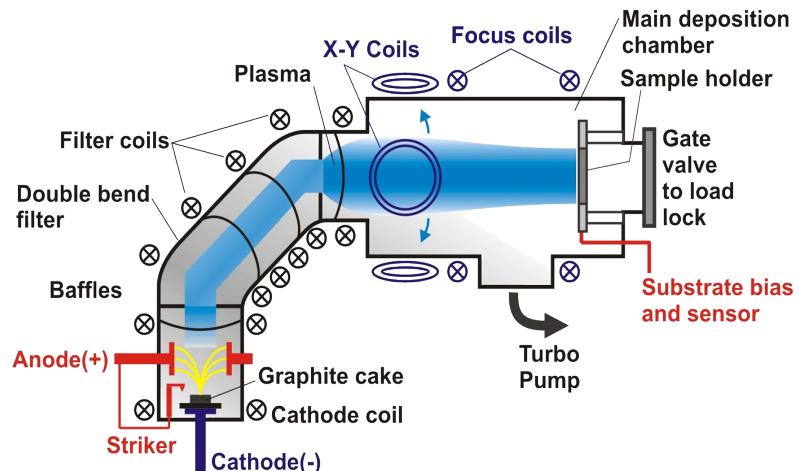
## The Ultrasmoothness of Diamond-like Carbon Surfaces

Michael Moseler,<sup>1,2\*</sup> Peter Gumbsch,<sup>1,3</sup> Cinzia Casiraghi,<sup>4</sup>  
Andrea C. Ferrari,<sup>4</sup> John Robertson<sup>4</sup>

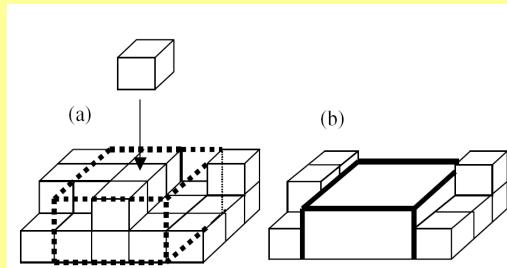
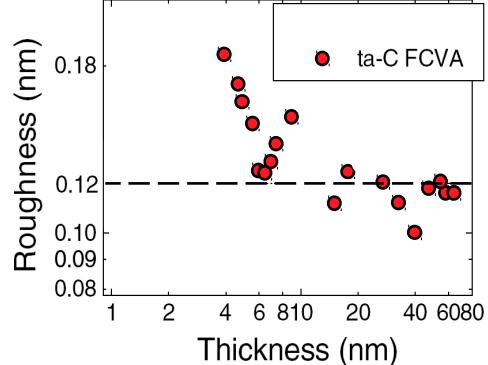
The ultrasmoothness of diamond-like carbon coatings is explained by an atomistic/continuum multiscale model. At the atomic scale, carbon ion impacts induce downhill currents in the top layer of a growing film. At the continuum scale, these currents cause a rapid smoothing of initially rough substrates by erosion of hills into neighboring hollows. The predicted surface evolution is in excellent agreement with atomic force microscopy measurements. This mechanism is general, as shown by similar simulations for amorphous silicon. It explains the recently reported smoothing of multilayers and amorphous transition metal oxide films and underlines the general importance of impact-induced downhill currents for ion deposition, polishing, and nanopatterning.

[www.sciencemag.org](http://www.sciencemag.org) SCIENCE VOL 309 2 SEPTEMBER 2005

1545

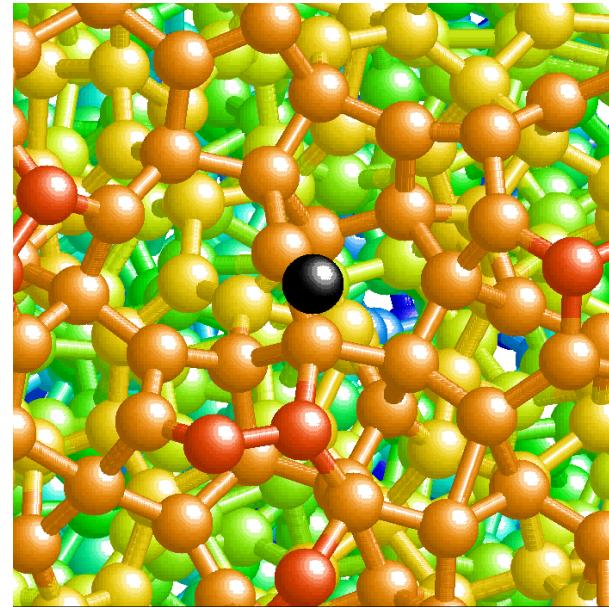


# Capillary smoothing?



Casiraghi, Ferrari, Ohr, Flewitt, Chu,  
Robertson, PRL **91**, 226104 (2003)

C impinges on ta-C  
with 100 eV



MD with Brenner BOP,  
D. W. Brenner. Phvs. Rev. B 42. 8458 (1990)

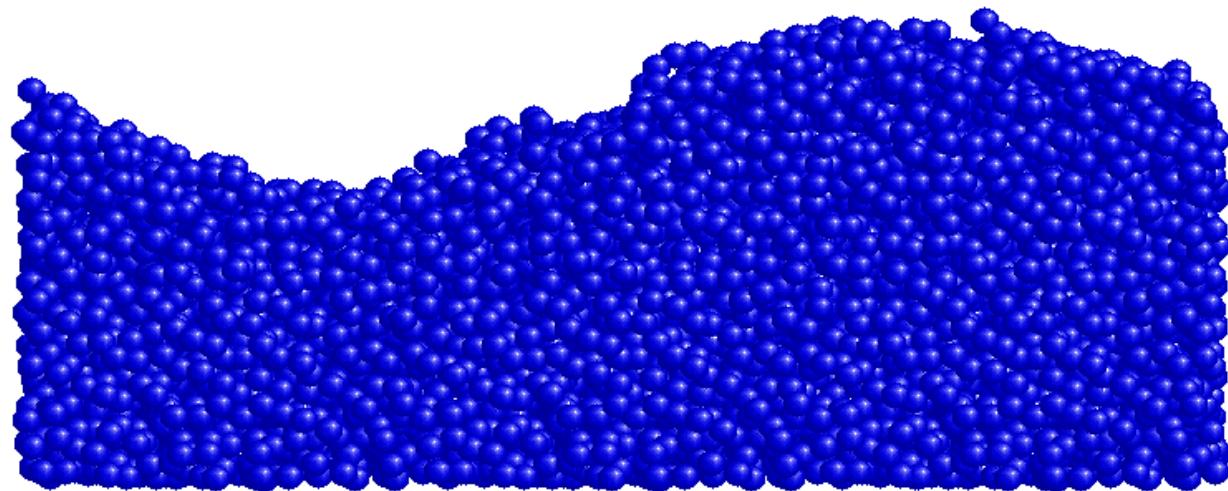
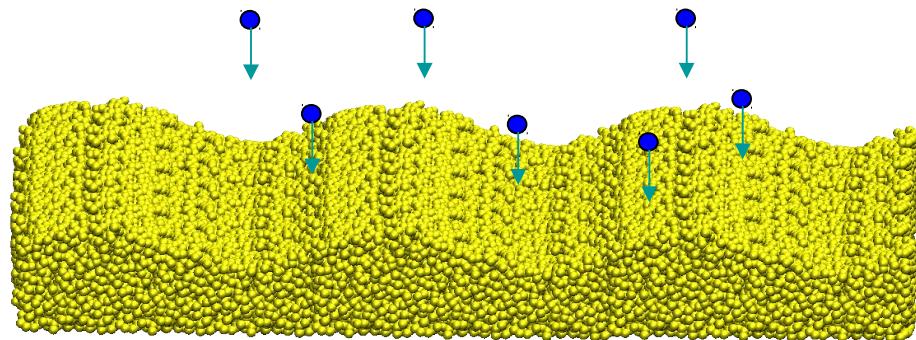
$$E_b = \sum_i \sum_{j(>i)} [V_R(r_{ij}) - \bar{B}_{ij} V_A(r_{ij})]$$



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DEPARTMENT OF ENGINEERING

# Atomistic simulation of film growth

The smoothing of  
a rough DLC film  
4000 C-atoms  
with 100 eV  
hit a film  
with an area  
7.05nm x 2.35nm



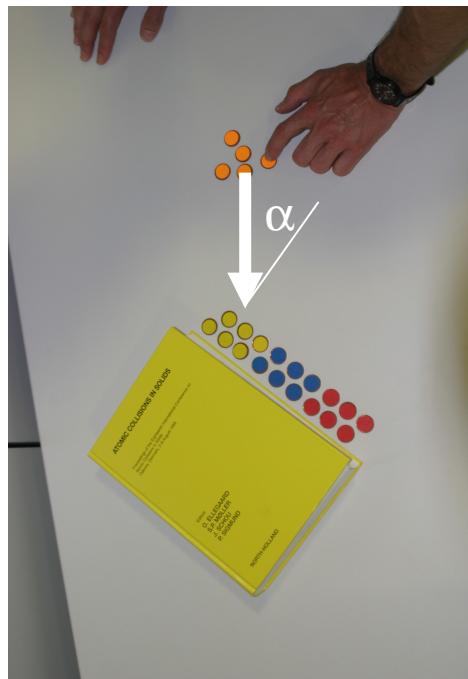
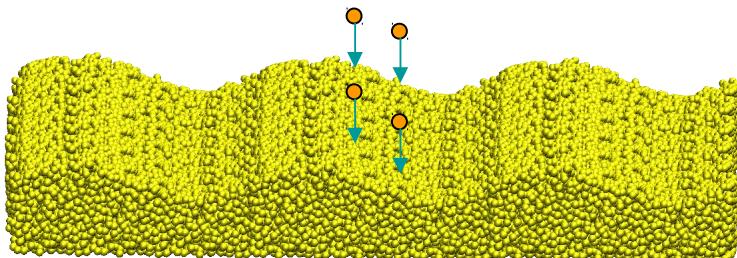
# Downhill currents

G.Carter, PRB 54,  
17647 (1996 )

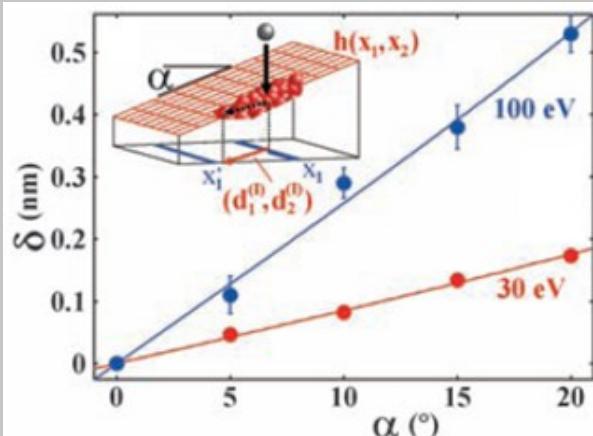
M.Moseler et al.

Comp. Mat. Sci. 10,  
452 (1998)

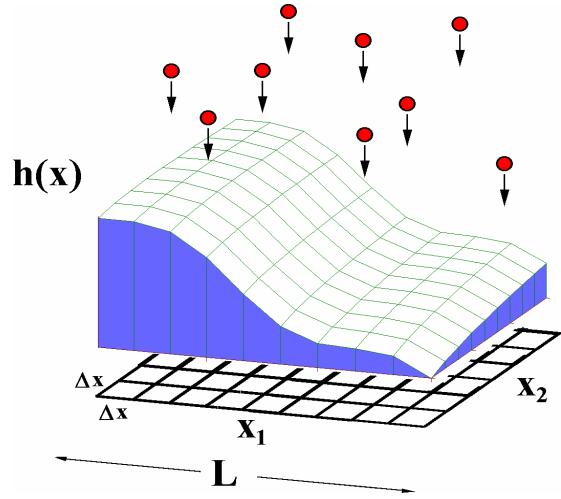
$h(x)$



Particle current:  
 $\mathbf{j}(\mathbf{x}) = -v \nabla h(\mathbf{x})$



# Mesoscale description



**Stochastic differential equation of motion**

$$\partial h(\mathbf{x}, t) / \partial t = -\Omega \nabla \cdot \mathbf{j}(\mathbf{x}, t) + \eta(\mathbf{x}, t)$$

$$\langle \eta(\mathbf{x}, t), \eta(\mathbf{x}', t') \rangle = r\Omega^2 \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

$$\mathbf{j}(\mathbf{x}, t) = -v \nabla h(\mathbf{x}, t)$$

**r: deposition rate, Ω: average atomic volume**

Stanley&Barabasi,  
Fractal concepts in Surface Growth

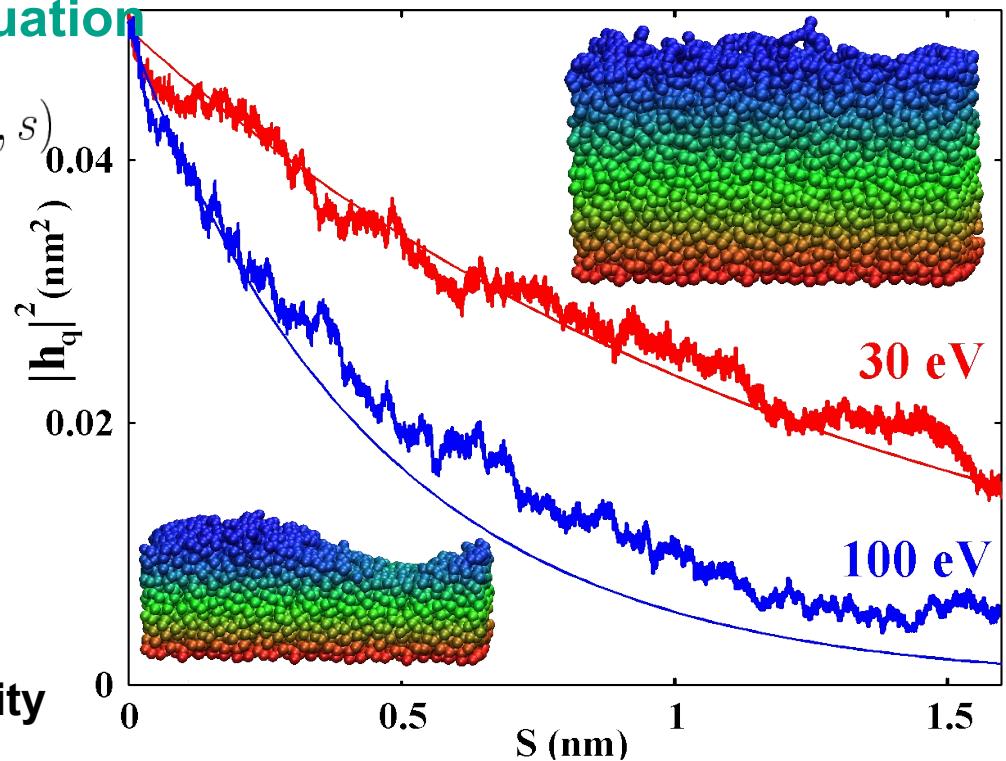
## The Edwards-Wilkinson equation

$$\partial h(\mathbf{x}, s) / \partial s = \nu \nabla^2 h(\mathbf{x}, s) + \eta(\mathbf{x}, s)$$

$s \sim t$

$$h(\mathbf{x}, s) \xleftarrow{FT} h_{\mathbf{k}}(s)$$

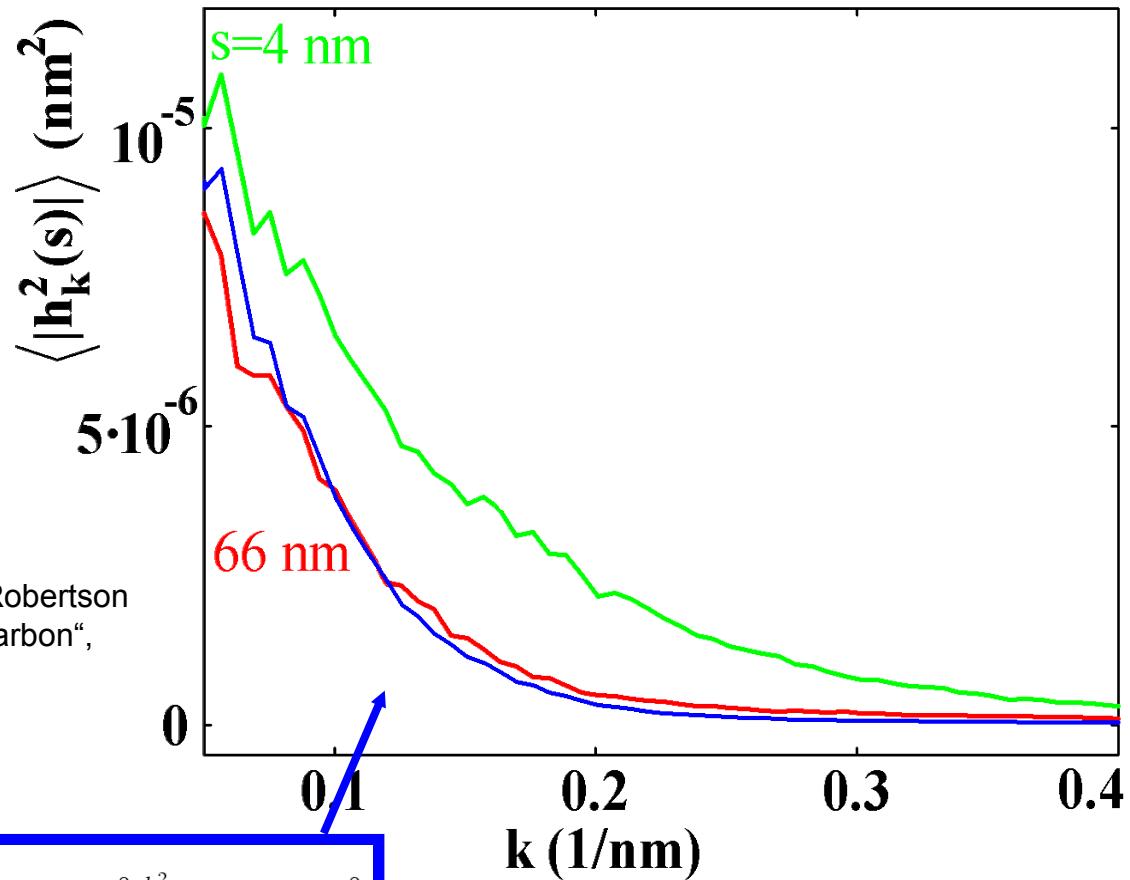
**Solution: Power spectral density**



$$\langle |h_{\mathbf{k}}(s)|^2 \rangle = e^{-2\nu k^2 s} \langle |h_{\mathbf{k}}(0)|^2 \rangle + \Omega(1 - e^{-2\nu k^2 s}) / (2\nu L_1 L_2 k^2)$$

## Evolution of the experimental power spectral density

Moseler, Gumbsch, Casiraghi, Ferrari, Robertson  
„The ultrasMOOTHNESS of diamond-like carbon“,  
Science **309**, 1545 (2005)



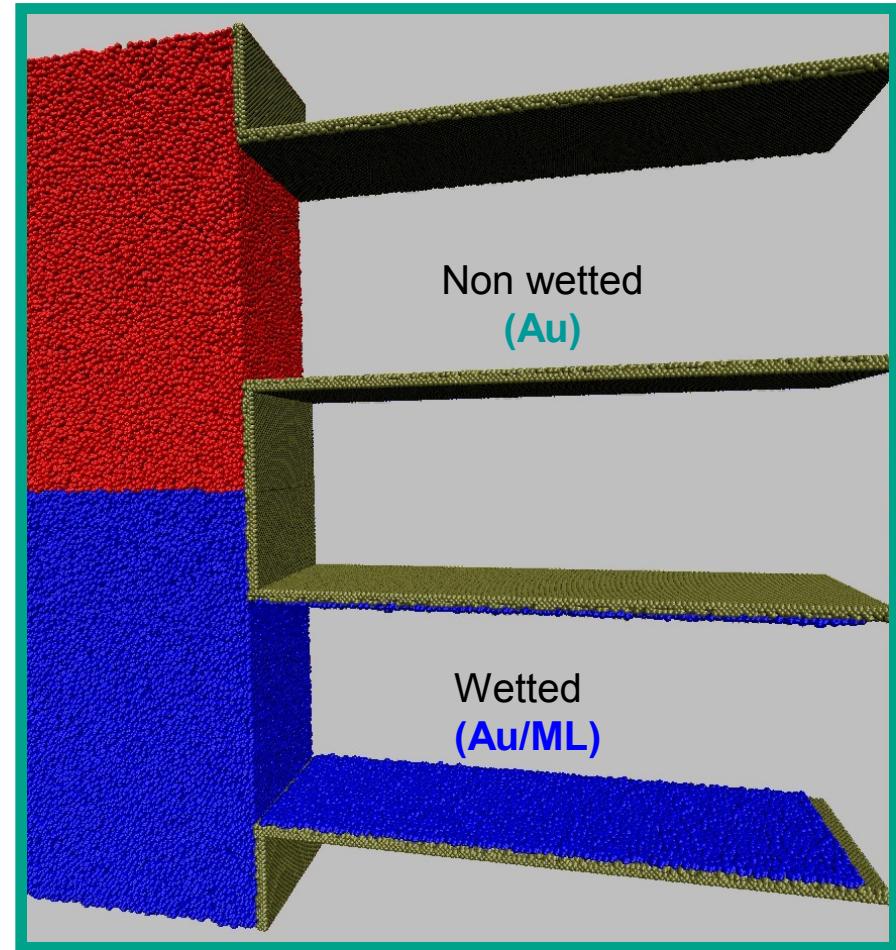
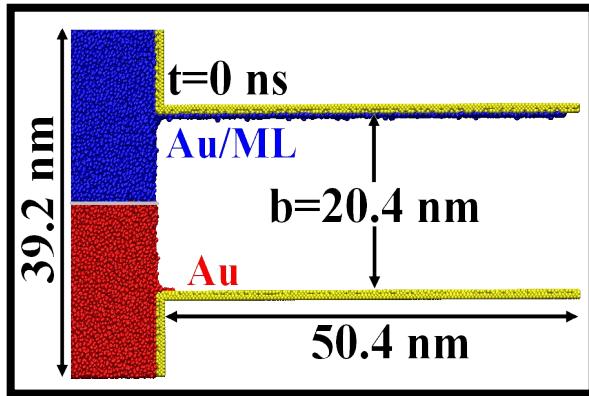
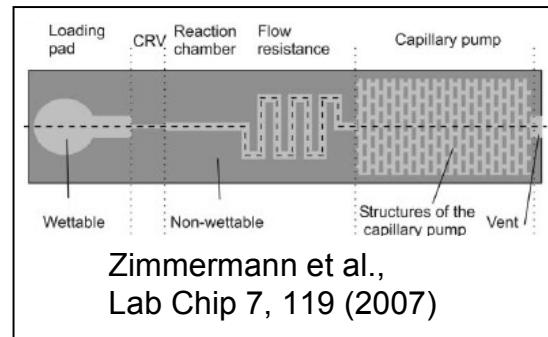
$$\langle |h_k(s)|^2 \rangle = e^{-2\nu k^2 s} \langle |h_k(0)|^2 \rangle + \Omega(1 - e^{-2\nu k^2 s}) / (2\nu L_1 L_2 k^2)$$

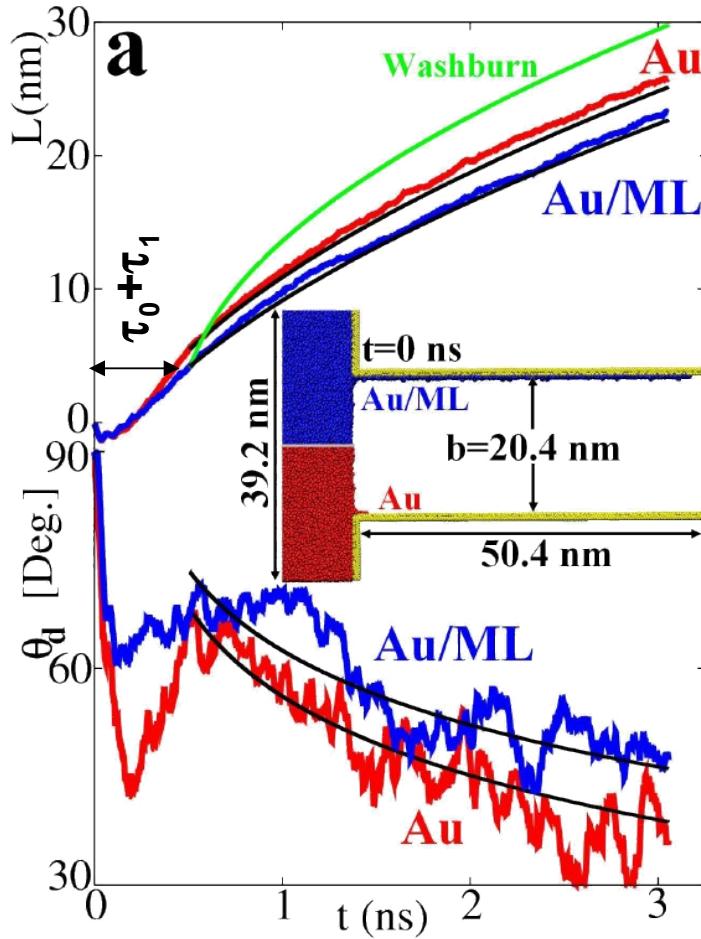


CAMBRIDGE UNIVERSITY  
DEPARTMENT OF ENGINEERING

# Nanocapillary pumps

- Applications in
- Lab-on-Chips
  - Nanotribology
  - Printing





Meniscus relaxation:

$$\tau_0 = \sqrt{\rho(b/2)^3/\gamma}$$

Establishment of Poiseuille flow:

$$\tau_1 = \rho(b/2)^2/\eta$$

D. Quere, Europhys.Lett. 39, 533 (1997).

**Washburn's law**

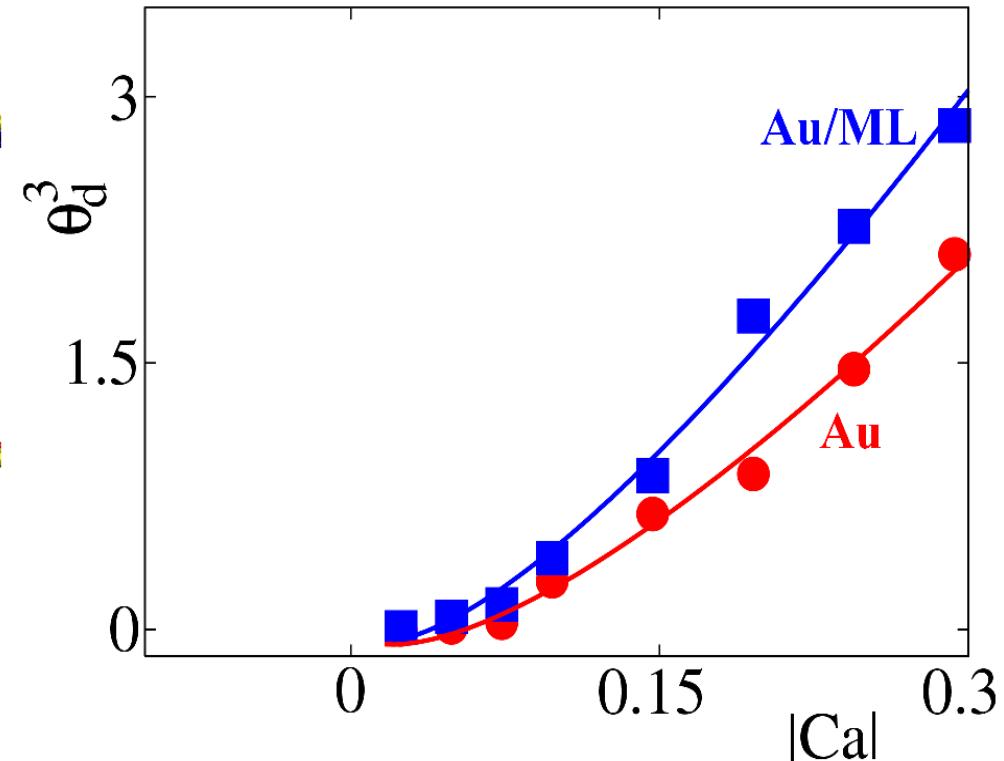
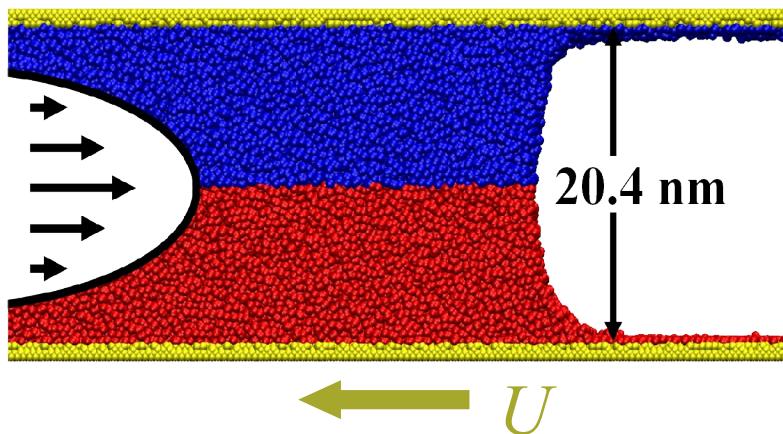
$$L(t) = \sqrt{\gamma b t \cos \theta_e / (3\eta)}$$

Balance of capillary and Poiseuille pressure:

$$p_c = \gamma \kappa$$

$$p_p = -12\eta L \dot{L} / b^2$$

## Steady state simulations

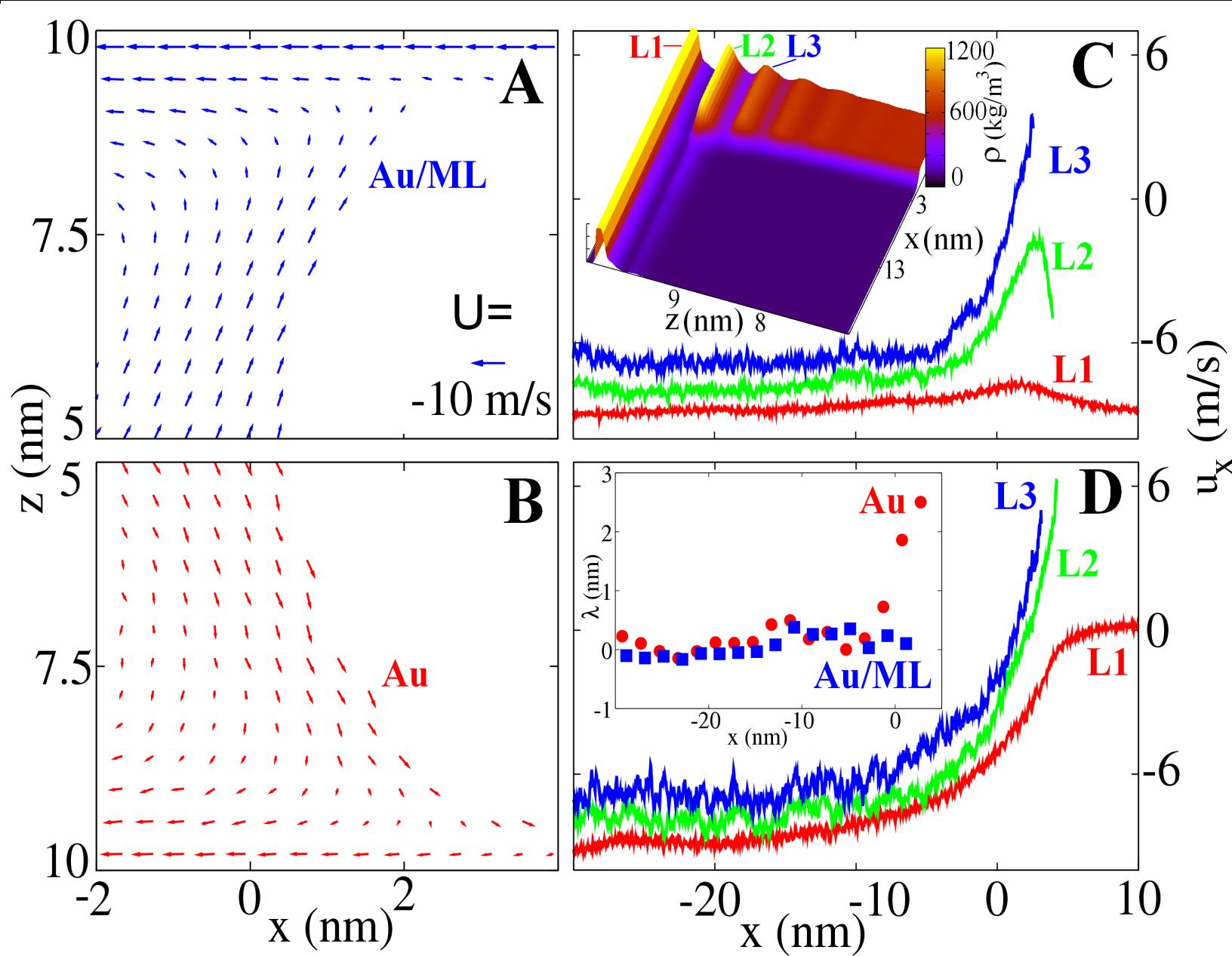


$$\text{Capillary Number: } Ca = \frac{\eta U}{\gamma}$$

$$\theta_d^3(Ca) = 9Ca \ln(\alpha Ca^\beta)$$

Au:  $\alpha = 5.97$  and  $\beta = 0.55$

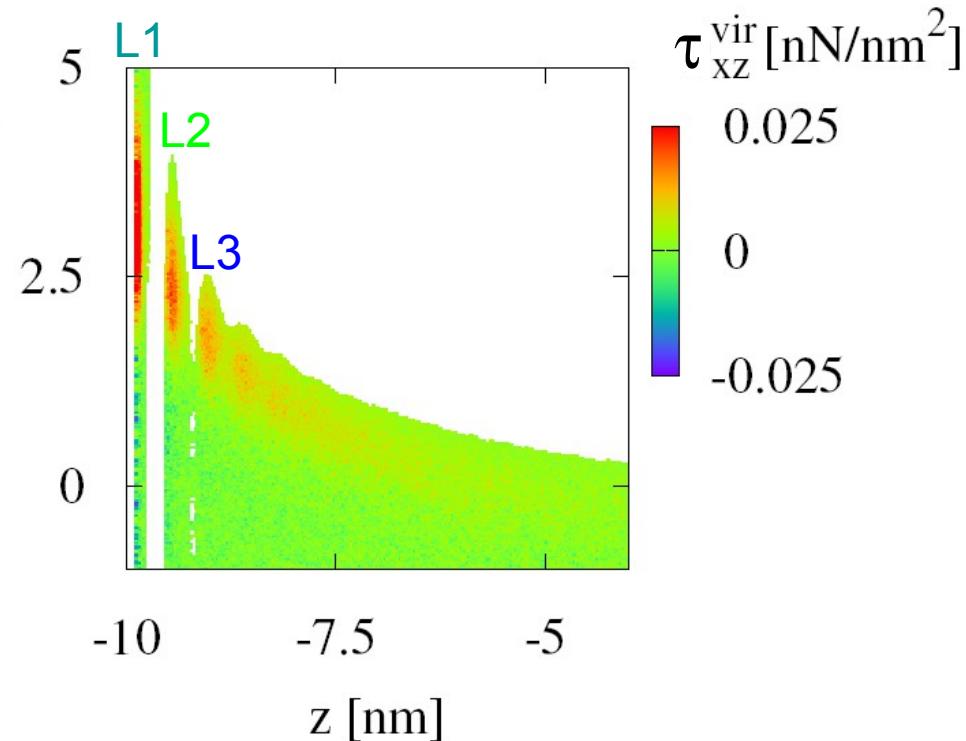
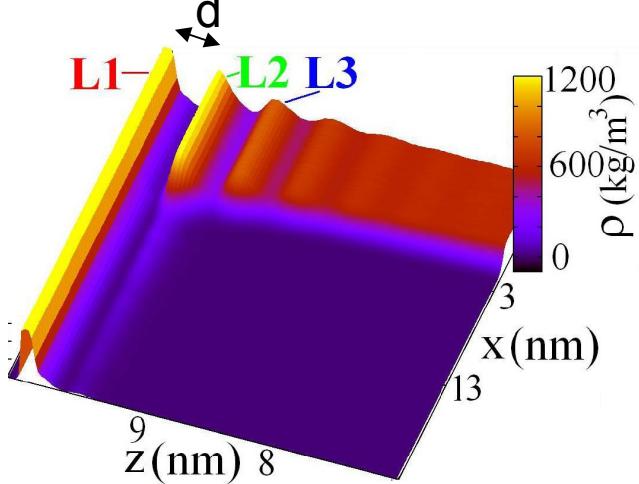
Au/ML:  $\alpha = 3.78$  and  $\beta = 0.46$



## The „stress singularity“:

$$\eta 2U/d = 0.0144 \text{ nN/nm}^2$$

$$\tau_{xz}^{vir} = \frac{1}{2V_C} \sum_i \sum_{j \neq i} x_{ij} (Fz)_{ij}.$$



# Continuum mechanical modelling

Velocity from lubrication approx.+ BCs

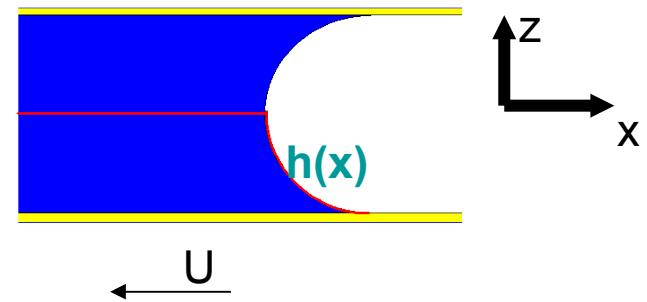
$$u_x(x, z) = U - [z^2 + 2h(x)z + 2h\lambda(x)] \partial_x p(x) / (2\eta)$$

Mass flux:

$$Q(t) = \int_0^h dz u_x = Uh - \partial_x p \left( \frac{h^3}{3} + \lambda h^2 \right) / \eta$$

Steady state:

$$Q = (U_{1L}^{wrf} + U)h_{1L}$$

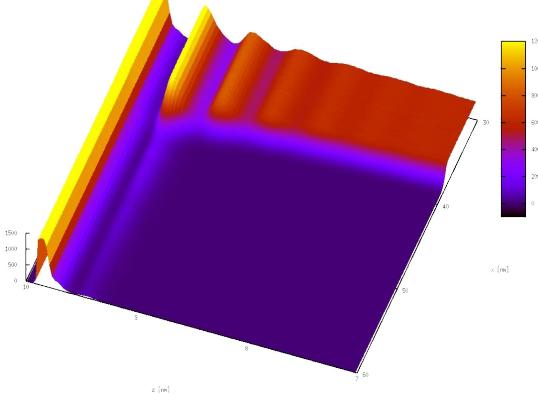
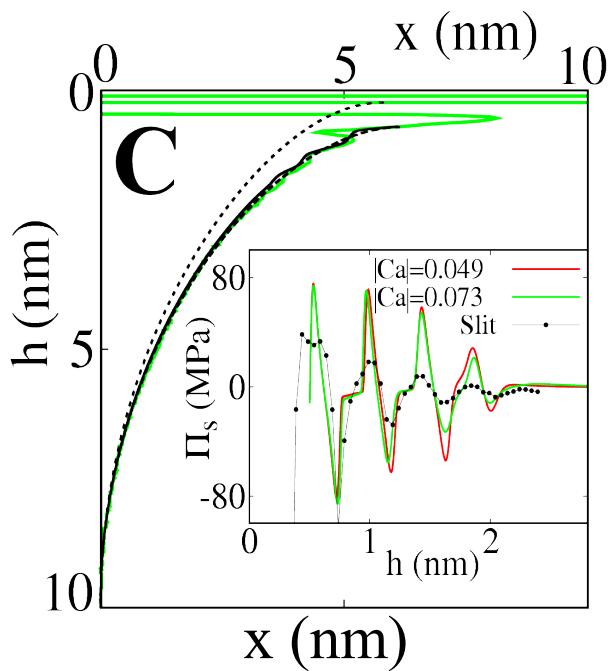
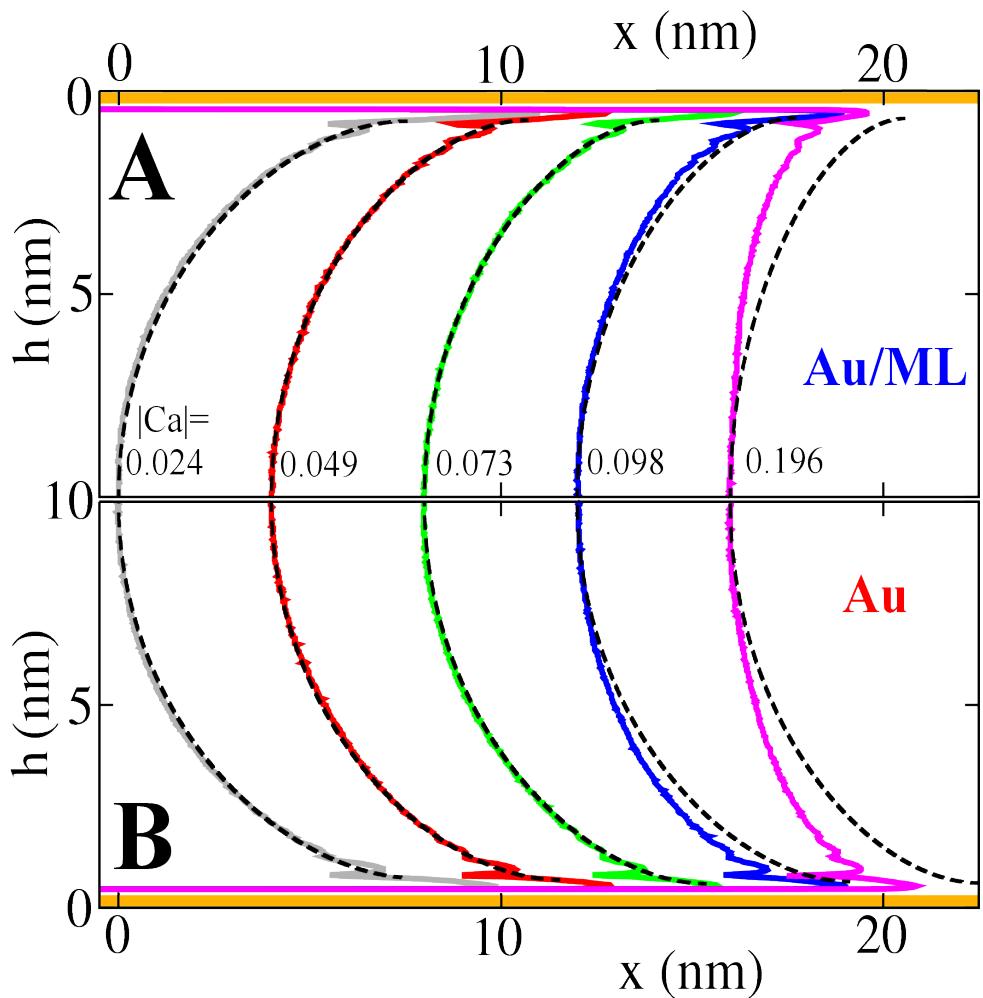


$$3 \frac{\text{Ca}(h - h_{1L}) - \text{Ca}_{1L}^{wrf} h_{1L}}{h^3} = \frac{\partial_x p}{\gamma} \left( 1 + \frac{3\lambda(x)}{h} \right)$$

$$\text{Ca}_{1L}^{wrf} = \frac{U_{1L}^{wrf} \eta}{\gamma}$$

$$\text{Au/ML:Ca}_{1L}^{wrf} = 0$$

$$\text{Au:Ca}_{1L}^{wrf} \approx 0.227$$



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$$\Pi_s(x) = \gamma \left( \int_0^x 3 \frac{\text{Ca}(h(\tilde{x}) - h_{1L})}{h^3(\tilde{x})} d\tilde{x} - \kappa \right)$$


---

## Extended Washburn equation

$$Q(t) = \int_0^h dz u_x = Uh - \partial_x p \left( \frac{h^3}{3} + \lambda h^2 \right) / \eta$$

$$Q(t) = \dot{L}b/2 \quad p_c = -2\gamma \cos \theta_d/b$$

