A Random Walk Through Search Research

Nick Watkins

NickWatkins@mykolab.com

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Thanks

- Rainer Klages for inviting me to participate in the ASG
- ONR for funding my participation in the ASG
- PKS for my visiting position in 2013-2014, hosted in Holger Kantz's group, and visits to this ASG in Dresden
- Ralf Metzler for hosting me at Potsdam during ASG

Overview I

- **Brownian paradigm:** keeps Central Limit Theorem, adapts walk models for movement & search research

- Continuous Time Random Walk & compound Brownian models as exemplar.

- **Non-Brownian paradigms:** arise from going to ECLT and/or modifying time dependence

- Levy Flights/Walks, Fractional Time Process as resulting exemplars

- Levy Foraging (or Environmental) Hypothesis as resulting conjectures

Overview II

- **My current interest:** Consequences of different ways of introducing time dependence-<u>either</u> Markovian <u>or</u> ergodic

- **Disclaimer:** First parts strongly influenced by my own experience in 2005-8 **[Edwards, Philips, Watkins et al, Nature, 2007]**. Hopefully, however, its semi-historical nature sets a stage for those who will go on to bring you up to date **[Zaburdaev, Bartumeus and others].**
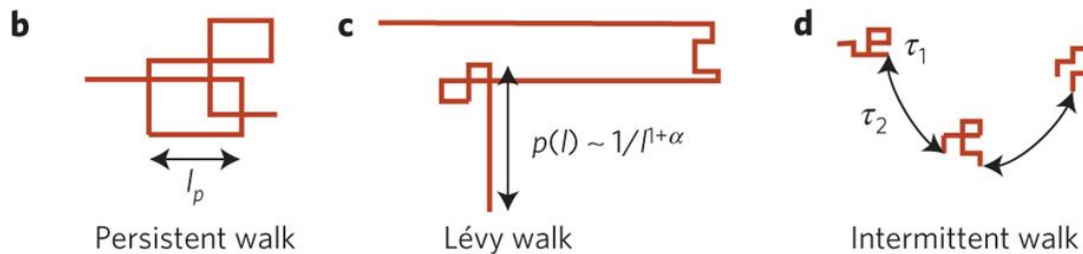
Movement modelling
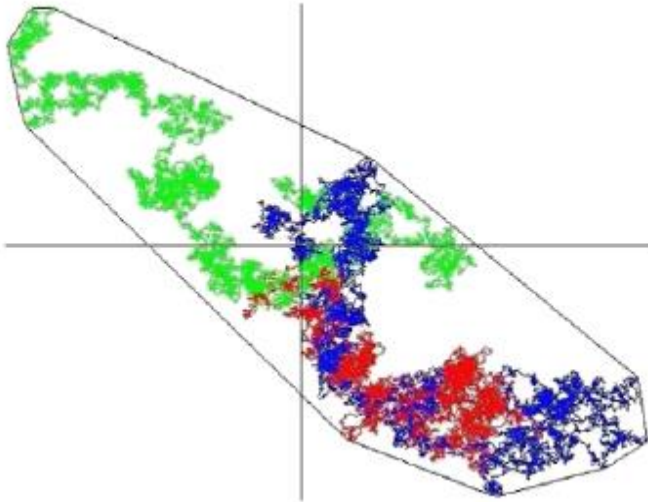
PATH INTELLIGENCE

- In widespread use, for very diverse reasons ...
- Home ranges of foraging animals **[e.g. Randon-Furling et al, PRL, 2009; Claussen et al, 2015]**
- Tracking shoppers in malls [e.g. **Path Intelligence**].

Search Research

THEORY

b Persistent walk

c Lévy walk
$p(l) \sim 1/l^{1+\alpha}$

d Intermittent walk
τ_1
τ_2

iRobot's Firstlook

PRACTICE

Chupeau et al, 2015

Why (and which) stochastic model?

- Movement patterns of biological organisms, including humans, very rich and complex.
- Fortunately, while reasonable to assume that animal movements deterministic on small spatiotemporal scales, on larger scales foraging paths exhibit random patterns so stochastic approaches to modelling exploited.
- But "randomness" comes in many flavours, how to choose right one for given case?

BROWNIAN PARADIGM

The connection of animal motion and random walks was made more than a century ago by pioneering statistician and biometrician **Karl Pearson.**

[Pearson, Biometric ser., 3, 54 (1906)].

Mathematical Contributions to the Theory of Evolution.

XV. A MATHEMATICAL THEORY OF RANDOM MIGRATION. By KARL PEARSON, F.R.S., with the assistance of JOHN BLAKEMAN, M.Sc.

(1) *Introductory.* In dealing with any natural phenomenon,—especially one of a vital nature, with all the complexity of living organisms in type and habit,— the mathematician has to simplify the conditions until they reach the attenuated character which lies within the power of his analysis*. The problem of migration is one which is largely statistical, but it involves at the same time a close study of geographical and geological conditions, and of food and shelter supply peculiar to each species. Some years ago the late Professor Weldon started an extensive study as to the distribution of various species and local races of land snails, but he was struck by the absence in several cases of any definite change of environment at the boundaries of the distribution of a definite race. It occurred to me in thinking over the matter that such boundaries, where they exist, may possibly not be permanent. To take a purely hypothetical illustration : A species is pushed back to a certain limit by a change of environmental conditions—say, an ice age. Does it follow that if the environment again becomes favourable, that it will *rapidly* occupy possible country ? What is the rate of infiltration of a species into a possible habitat ? It depends, of course, on a whole series of most complex conditions, the rate of locomotion, the channels of communication, the distribution of food areas and breeding grounds in the new country, and the connecting links between all these. Every detail must be studied by the field naturalist in relation to each species. All the mathematician can do is to make an idealised system, which may be dangerous, if applied dogmatically to any particular case, but which can hardly fail to be suggestive, if it be treated within the limits of reasonable application. The idealised system which I proposed to myself was of the following kind :

(i) Breeding grounds and food supply are supposed to have an average uniform distribution over the district under consideration. There is to be no special following of river beds or forest tracks.

* This is of course a perfectly familiar process to every mathematical physicist, but its unfamiliarity leads the biologist to suspect or even discard mathematical reasoning, instead of testing the result as the physicist does by experiment and observation.

1—2

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- **Pearson** also coined the term *random walk* when posing it as a problem in a famous letter to Nature, which was answered by **Lord Rayleigh** who showed the Gaussian nature of the solution and linked it to a problem he had solved in physics.

- As the theory of Brownian random walks and the Wiener process rapidly developed in statistical physics and mathematics, this concept came to dominate the stochastic modeling of movement in biology.

4   KARL PEARSON

(ii) The species scattering from a centre is supposed to distribute itself uniformly in all directions. The average distance through which an individual of the species moves from habitat to habitat will be spoken of as a "flight," and there may be n such "flights" from locus of origin to breeding ground, or again from breeding ground to breeding ground, if the species reproduces more than once. A flight is to be distinguished from a "flitter," a mere two and fro motion associated with the quest for food or mate in the neighbourhood of the habitat.

(iii) Now taking a centre, reduced in the idealised system to a point, what would be the distribution after n random flights of N individuals departing from this centre? This is the *first* problem. I will call it the *Fundamental Problem of Random Migration*.

(iv) Supposing the first problem solved, we have now to distribute such points over an area bounded by any contour, and mark the distribution on both sides of the contour after any number of breeding seasons. The shape of the contour and the number of seasons dealt with provide a series of problems which may be spoken of as *Secondary Problems of Migration*.

A little consideration of the Fundamental Problem showed me that it presented considerable analytical difficulties, and I was by no means clear that the series of hypotheses adopted would be sufficiently close to the natural conditions of any species to repay the labour involved in the investigation. At this stage the matter rested, until last year Major Ross put before me the same problem as being of essential importance for the infiltration of mosquitoes into cleared areas, and asked me if I could not provide the statistical solution of it. He considered that we might treat a district as approximately "equi-swampous," and thus my conditions (i), (ii) above could be applied to obtain at any rate a first approximation to the solution.

Starting on the problem again I obtained the solution for the distribution after two flights, an integral expressing the distribution after three flights, which I carelessly failed to see could be at once reduced to an elliptic integral, and the general functional relation between the distribution after successive flights. At this point I failed to make further progress, and under the heading of "The Problem of the Random Walk" asked for the aid of fellow-mathematicians in *Nature**. The reply to my appeal was threefold. Mr Geoffrey T. Bennett sent me in terms of elliptic integrals the solution for three flights. Lord Rayleigh drew my attention to the fact that the "problem of the random walk" where the number of flights is very great becomes identical with a problem in the combination of sound amplitudes in the case of notes of the same period, which he has dealt with in several papers†. Lastly Professor J. C. Kluyver presented a paper to the Royal Academy

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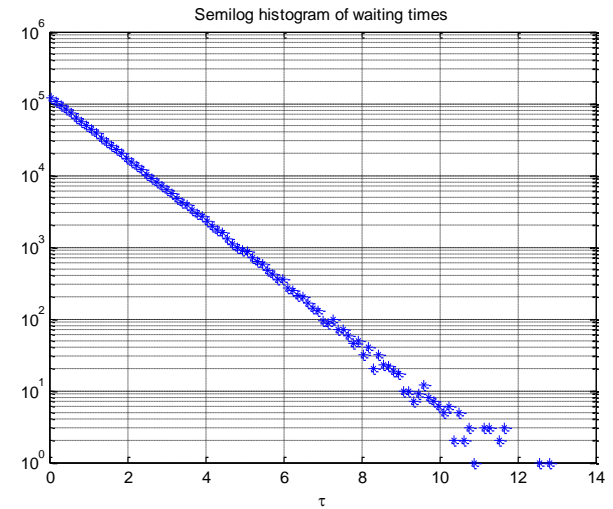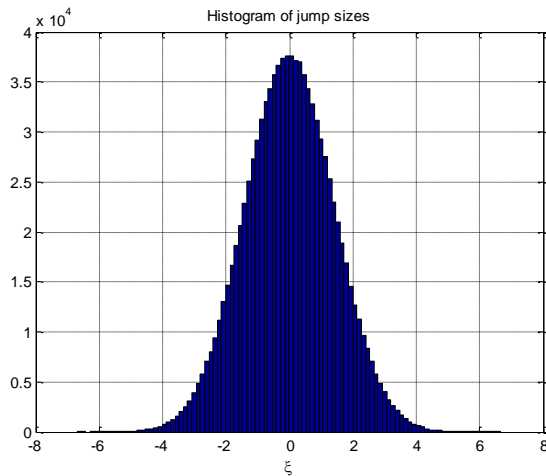
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CTRW as archetype

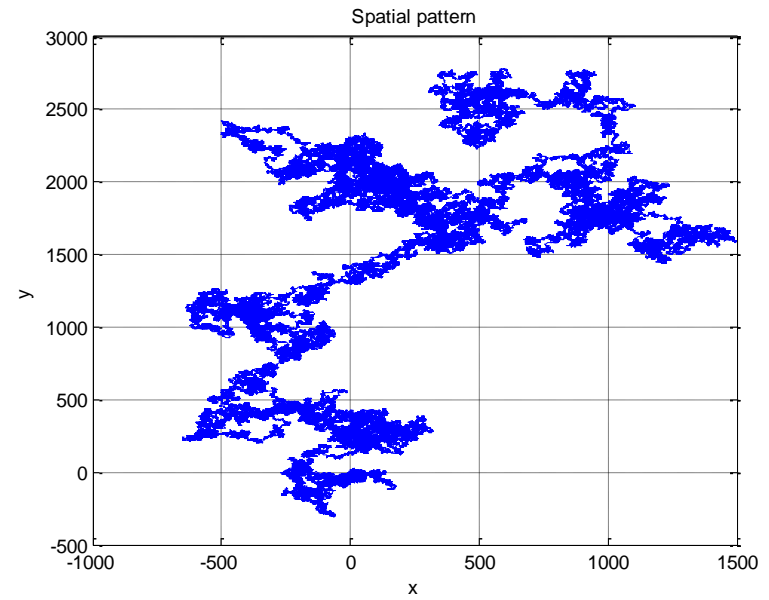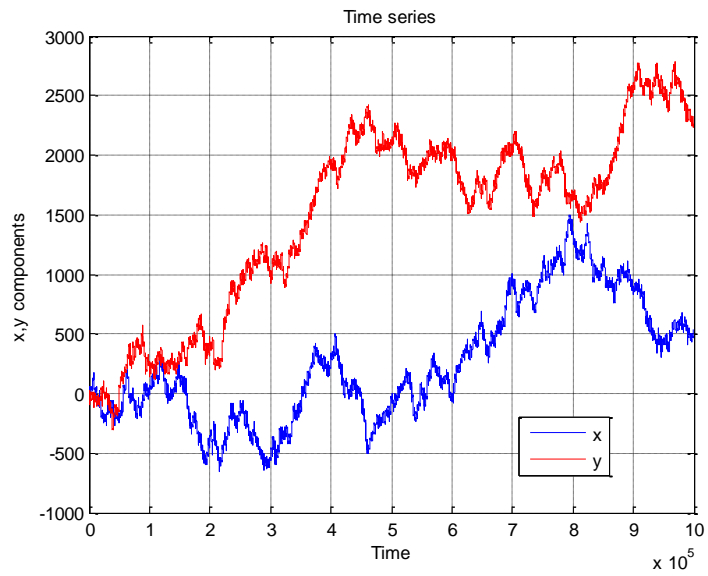$$\varphi(\xi, \tau) = \lambda(\xi)\psi(\tau)$$

Histogram of jump sizes

Semilog histogram of waiting times

$$X(t) = \sum_{i=1}^{n} \xi_i \qquad \gamma_x = (\gamma_\tau)^{\beta/\alpha} \qquad t_n = \sum_{i=1}^{n} \tau_i$$
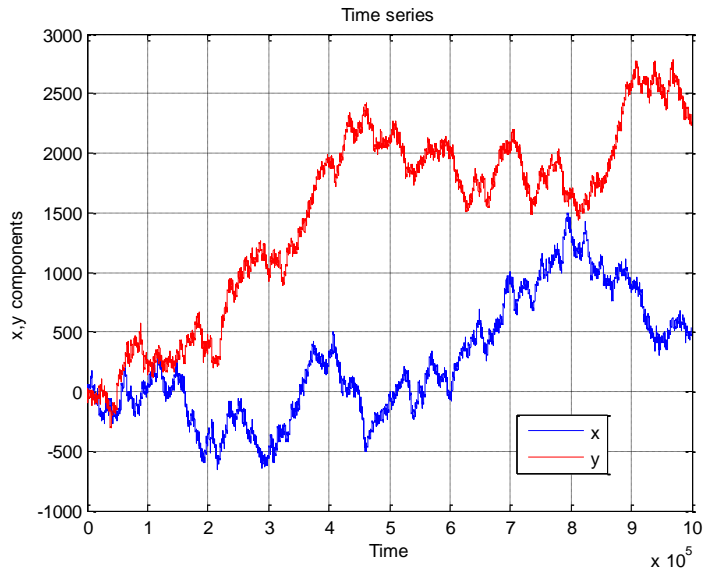
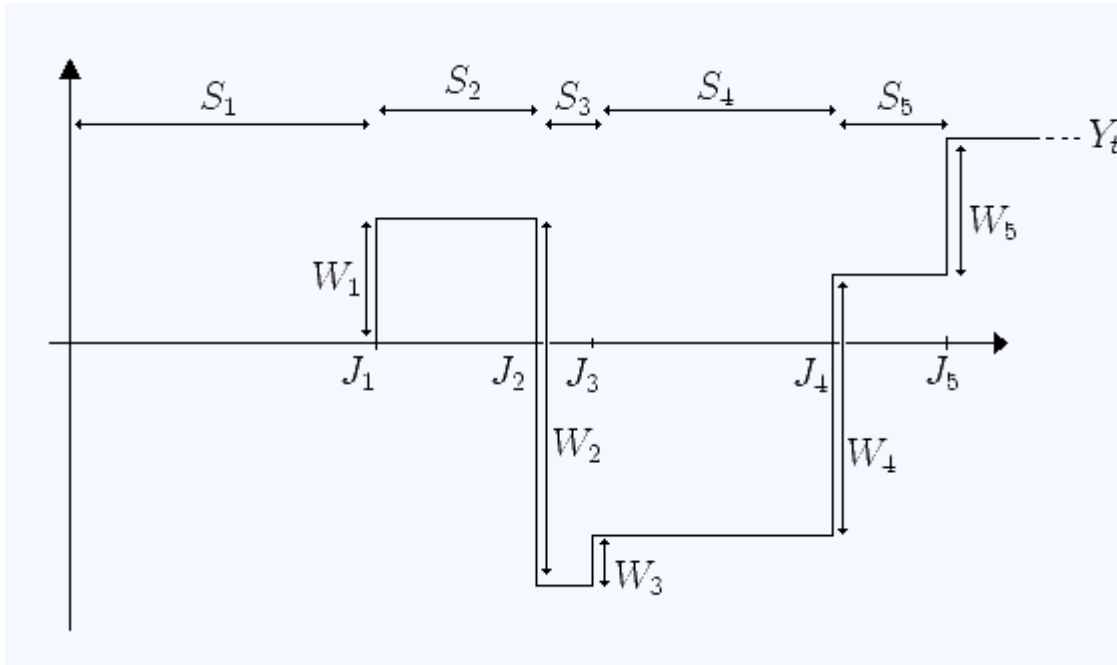Notation as Fulger *et al*, PRE (2008)

2D CTRW

2D CTRW

Time series

Spatial pattern
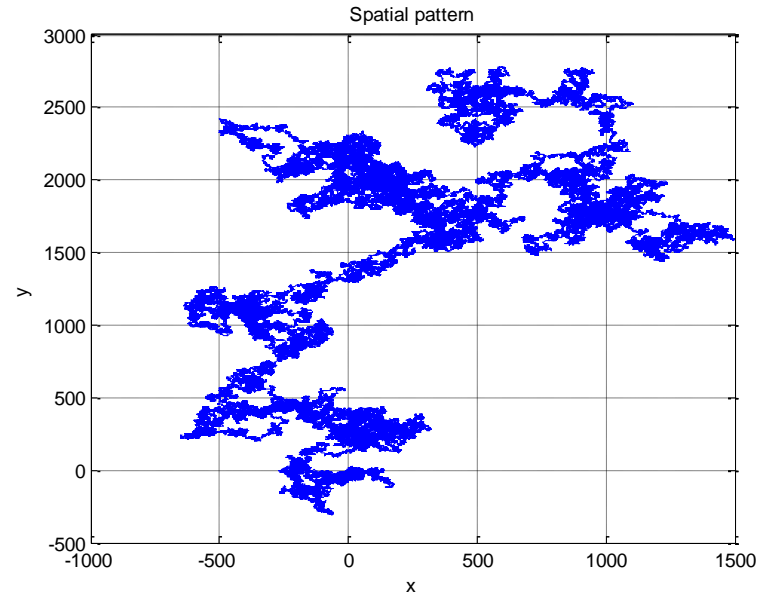
Another choice would
be single random vector
length R with uniform
random angle θ
– which choice is adopted
becomes more important for
non-Brownian cases

CTRW = renewal reward process

Here jumps at {J} become rewards {W} and waiting times become holding times {S}

Much scope for interplay of maths and physics

Compound Brownian ?

- As measurement techniques in experimental biology advanced, was found that ordinary Brownian motion too simple to generally represent foraging paths.

- Variations on simple random walk paradigm, notably composite correlated random walks and complex intermittent search strategies combining scanning and relocation modes developed and applied to explain biological movement data with much success **[e.g Benhamou, 2014; Benichou et al, 2011; Campos et al, 2014].**

- All generate "normal diffusion" in long time limit, mean squared displacement of ensemble of foragers grows linearly in time.

b

l_p

Persistent walk

d

τ_1

τ_2

Intermittent walk
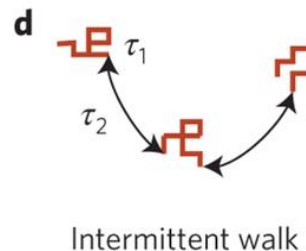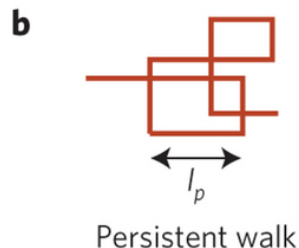
BEYOND BROWNIAN MOTION

Symptoms of complex transport: 1

Possibility of very long jumps
("flights") compared to the <jump>

Anomalous diffusion ?

- However, at least one observed feature of animal foraging (& human movement) motivates something other than Brownian, or even compound Brownian, picture.

- Many individuals across a broad range of species observed to make small local random movements interspersed with rare long-distance jumps. Particularly true of affluent modern humans …

- In consequence Brownian theoretical framework for diffusion in both biology and the physical sciences has been challenged over the past three decades by a new emerging synthesis.

- This is drawn both from data and from the theory of those stochastic processes which generate non-Brownian *"anomalous" diffusion*, where the mean squared displacement grows <u>nonlinearly</u> in the long time limit.

ECLT
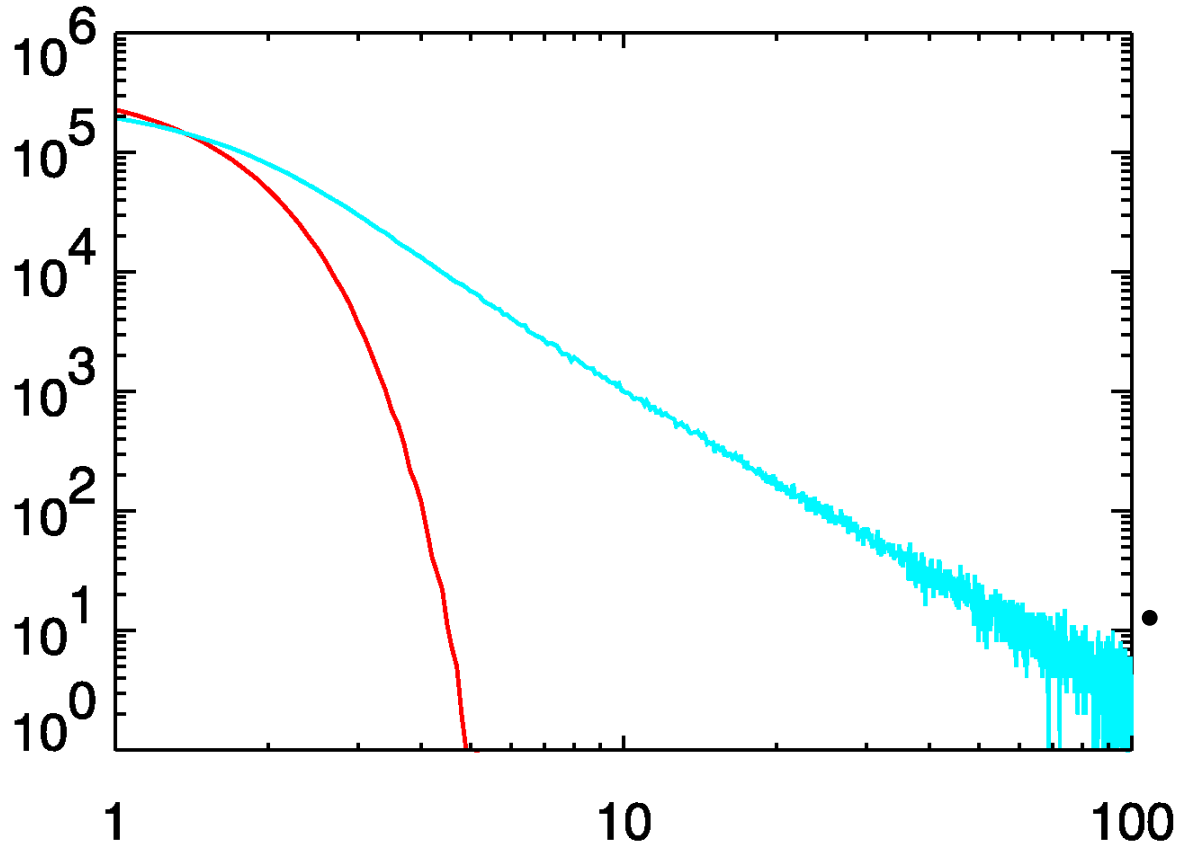
$$\lambda(\xi) = \xi^{-(1+\alpha)} \qquad 0 < \alpha < 2$$

- Brownian motion paradigm for random walks was closely linked to central limit theorem (CLT), <u>but</u> known for much of the last century that relaxing just one of assumptions-finite variance-gives a new class of random walk.

- This class obeys extended central limit theorem-has a very different probability distribution for the size of a step-asymptotic power law tail.

Levy flights

- The resulting stochastic movement model is called a *Lévy flight*. Intriguingly, like the biological observations, a "Lévy flyer" has apparent clusters linked by long jumps.
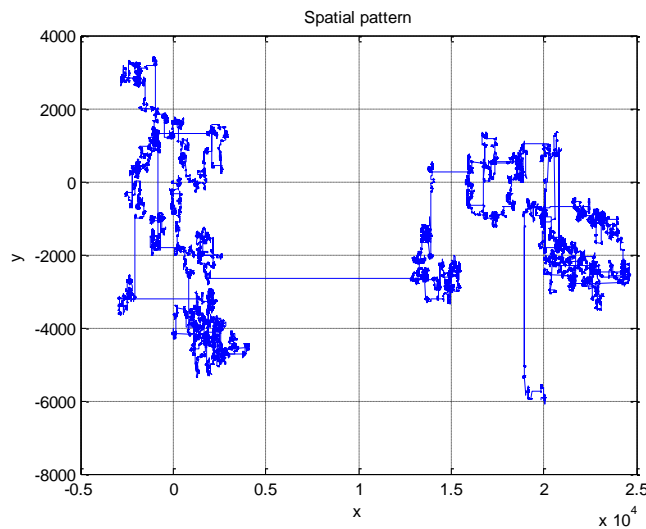
α=1.5, β=1
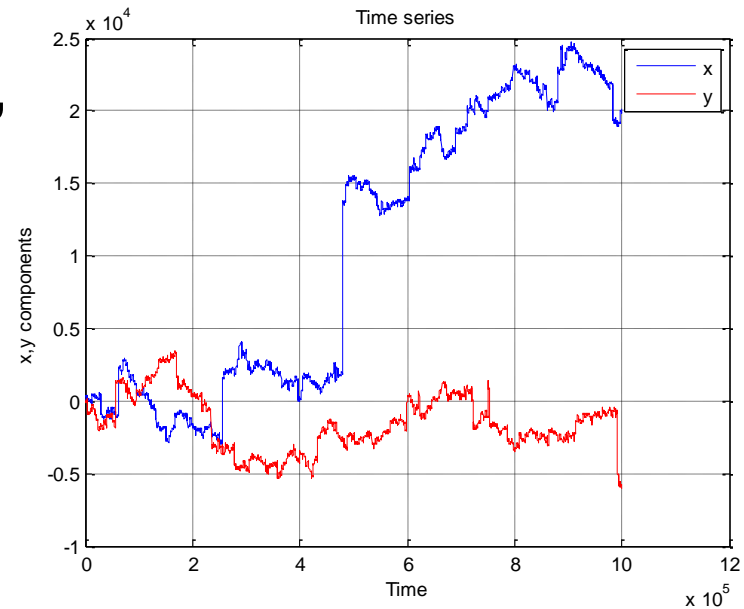
Symptoms of complex transport: 2

... longer waiting times

24 August 2010 Last updated at 00:47

China traffic jam stretches 'nine days, 100km'

A massive traffic jam in China has slowed vehicles to a crawl for nine days near Beijing, local media say.

Vehicles, mostly lorries bound for Beijing, are in a queue for about 100km (62 miles) because of heavy traffic, road works and breakdowns.

The drivers have complained that locals were over-charging them for food and drink while they were stuck.

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The situation has now "basically returned to normal," state television said on Monday.

There has been a boom in road building in China in recent years but vehicle use has soared at the same time.

The stalled traffic stretched between Jining in Inner Mongolia and Huai'an in Hebei province, north-west of Beijing, said the Global Times.

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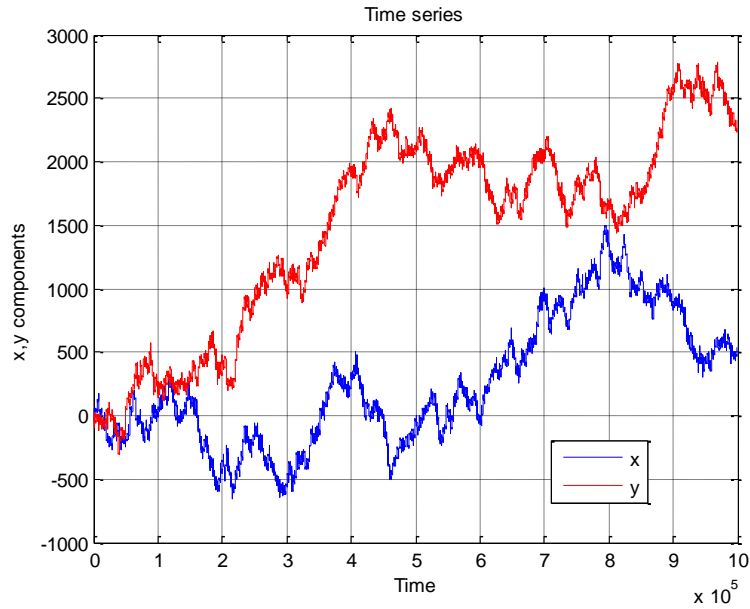The drivers say locals are over-charging them for food and drink

Fractional Time Process

- Another type of anomalous diffusion-what you get if you keep Markovian structure and factorising probability but allow long tails in waiting time between jumps.
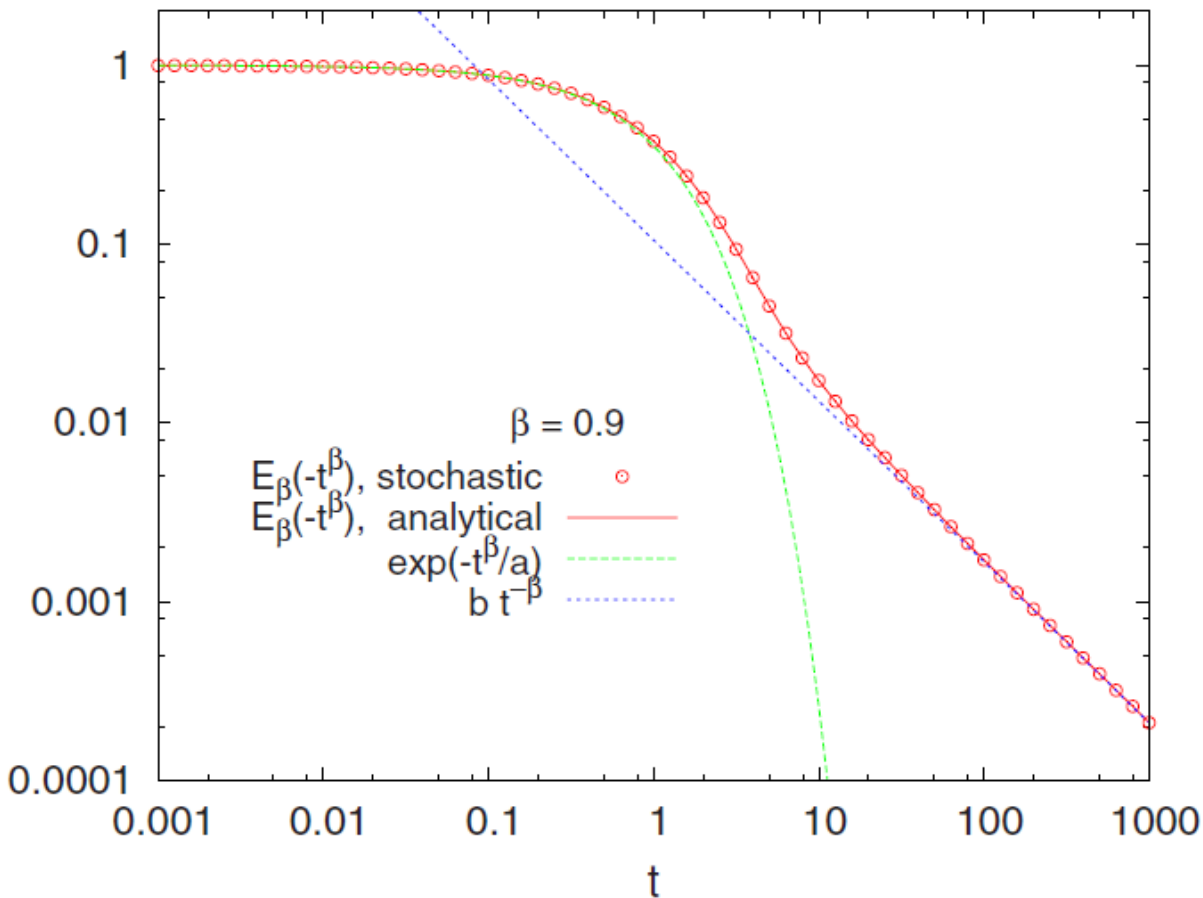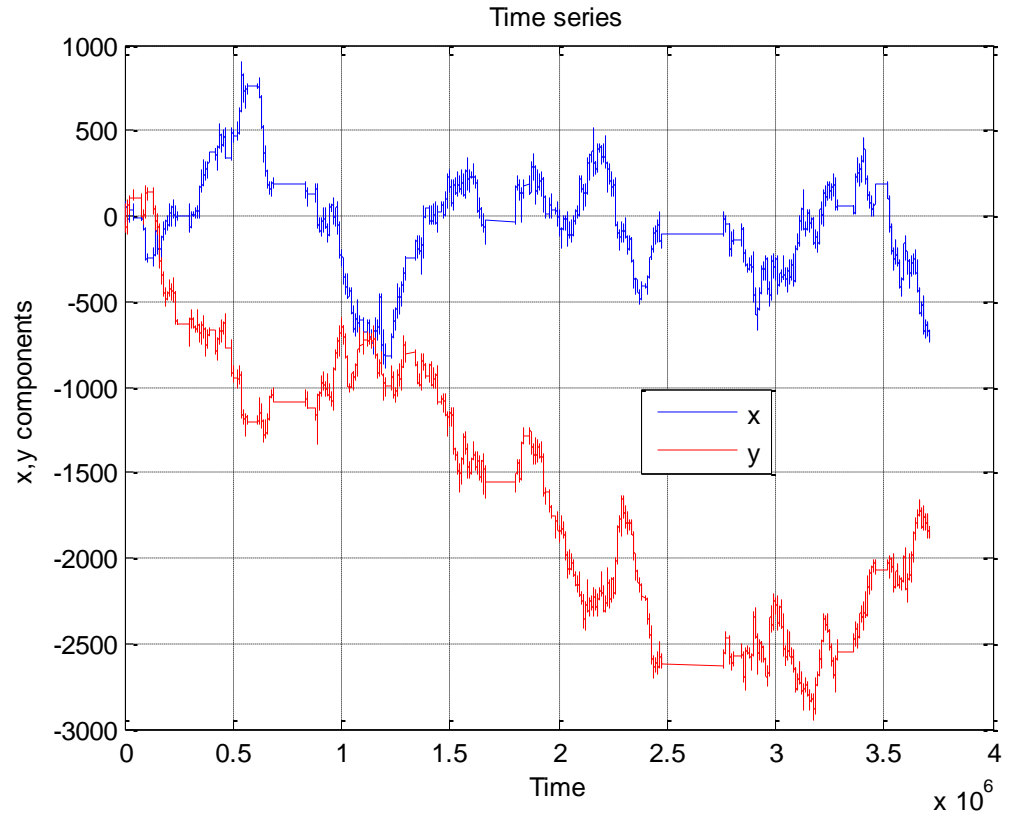
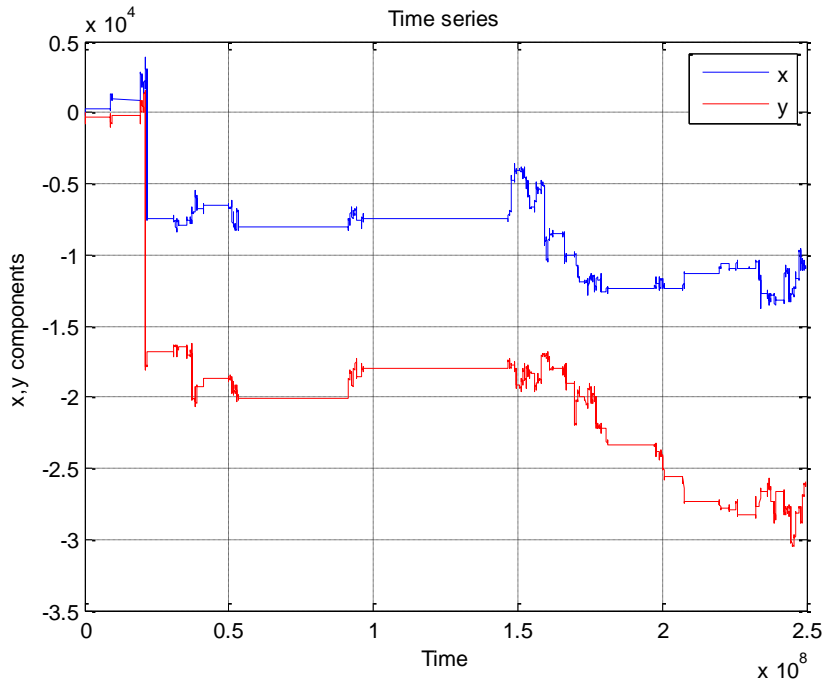Example here is **Mittag-Leffler,** has advantage that one limit is exponential

FTP Time series
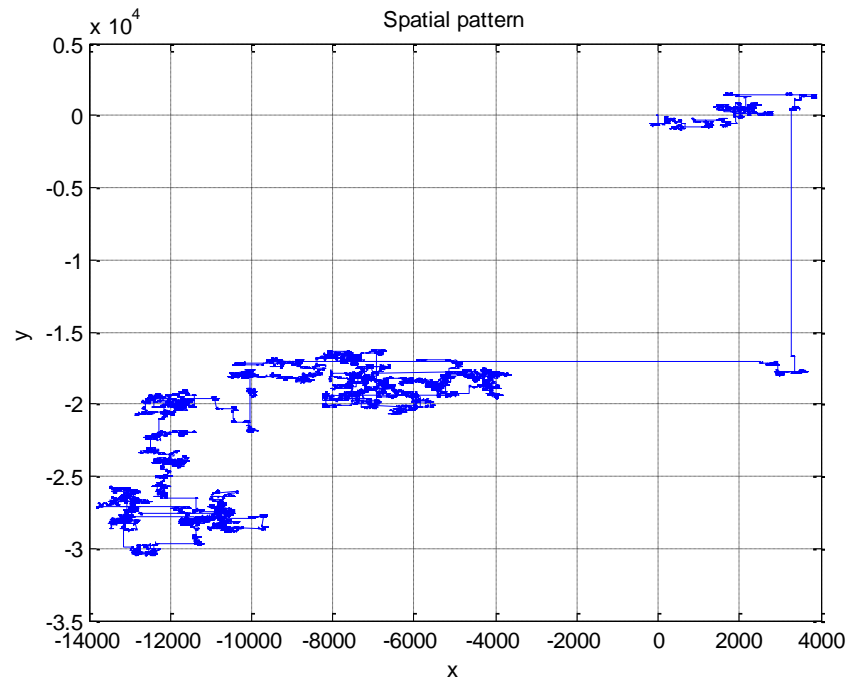

Brownian example for comparison

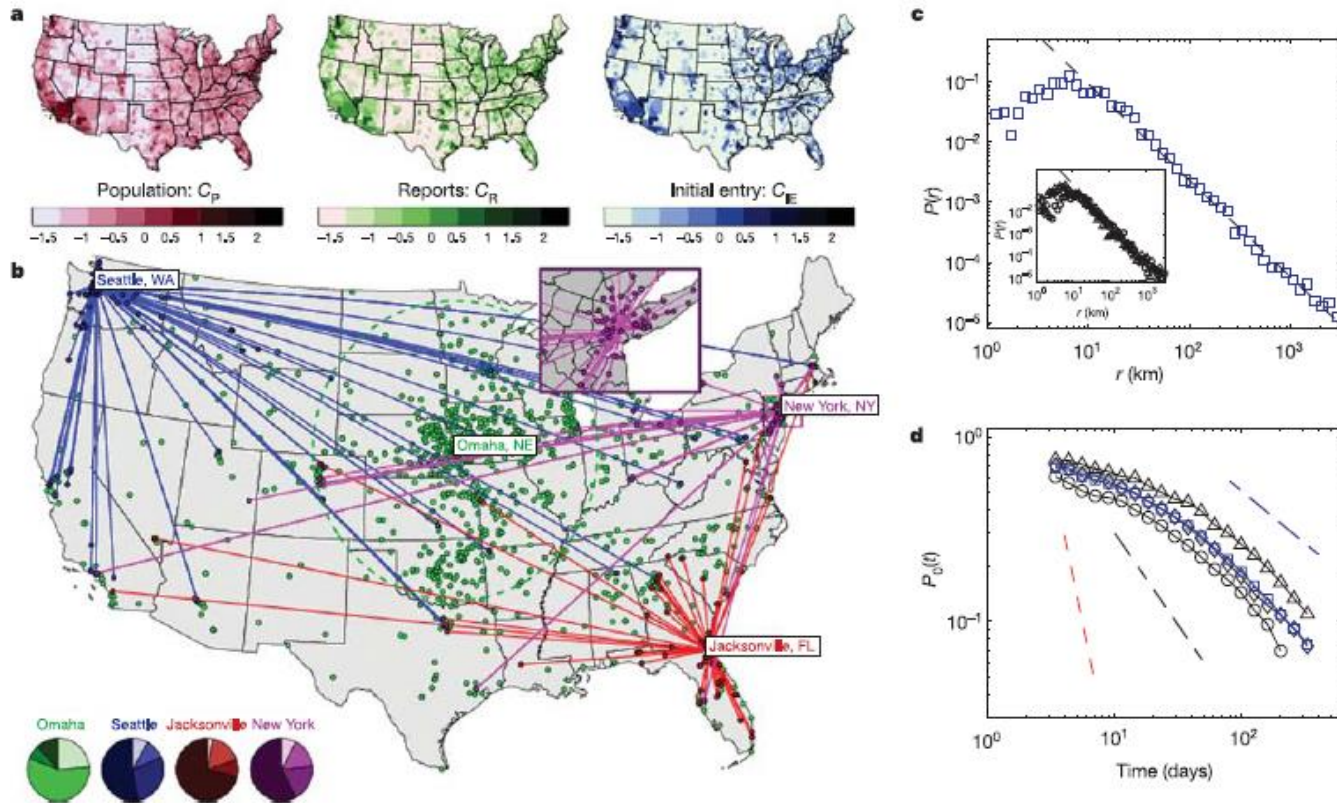α=2, β= 0.9

Can Combine LF & FTP

Called "Fully Fractional CTRW"

time series

space

α=1.5, β=0.7
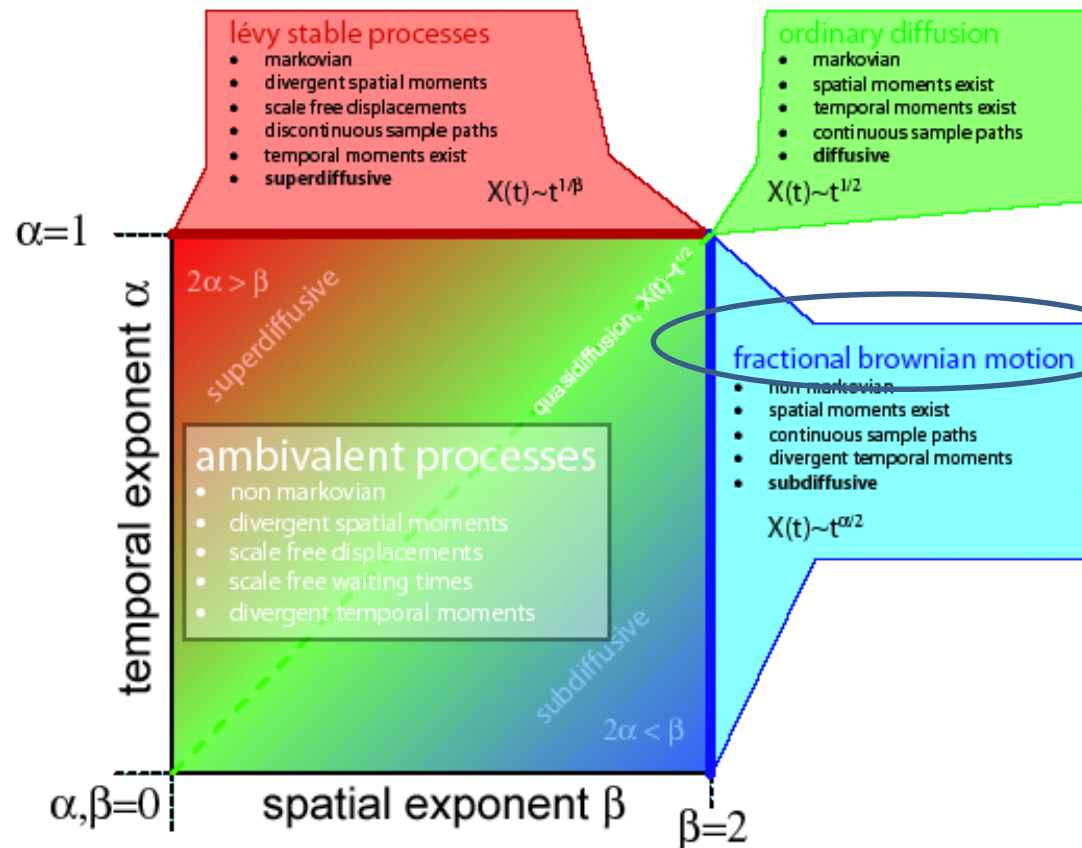
Dollar bills [Brockmann, 2006]

Data fitted to an FFCTRW with α, β both about 0.6

See also **Bartumeus, Giuggioli et al's work on Shearwaters, 2010**

Can Combine LF & FTP

From supp. Info. of [Brockmann et al, 2006] comes very useful schematic-

NB they defined (α,β) opposite way to Fulger and others.

Actually <u>not</u> fBm, but rather in fact the fractional time process (FTP).

Don't believe everything you read in Nature ;-)

Another way: <u>couple</u> space to time

Gives a finite velocity by introducing a jump duration τ' & coupling the jump size to it – idea known as Lévy <u>walk</u>
[Shlesinger & Klafter, PRL (1985)].

Lévy walk

$$\varphi(\xi, \tau') = \Psi(\tau' \,|\, \xi)\lambda(\xi)$$

$$= \delta(|\xi| - \tau')\lambda(\xi)$$

Uncoupled CTRW

$$\varphi(\xi, \tau) = \lambda(\xi)\psi(\tau)$$

In above τ' means flight <u>duration</u> in Levy walk, and τ <u>waiting time</u> in CTRW.

Viswanathan et al 1996 [V96]
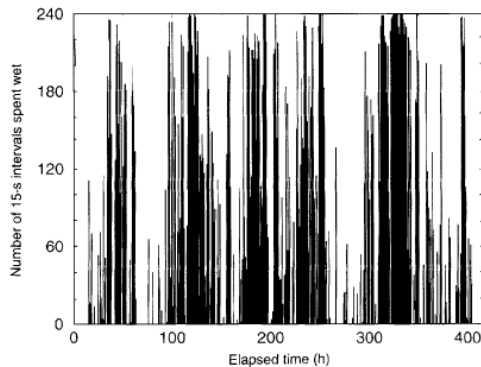

LETTERS TO NATURE

FIG. 1 The longest of the 19 time series, with a length of 416 h. Each point in the time series gives the number of 15-s intervals in each hour for which the animal was wet for 9 s or more.
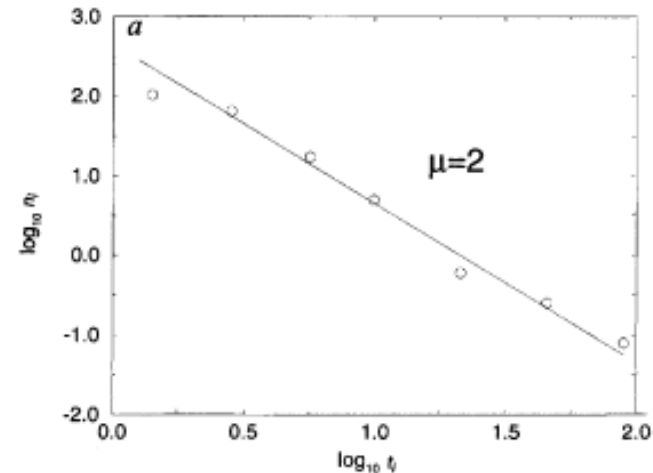

$\mu=2$

FIG. 3 a, Distribution giving the number n_i of intervals in the entire data set with flight-time intervals t_j. We used bin widths of 2^k h for the bin k, and used the geometric midpoints of the bins to plot the results. The Lévy-walk model of foraging fits the data quite well, as can be seen by the agreement of the data with the straight line of slope −2. b, Double log plot of the r.m.s.

"In the days of sail the bird often accompanied ships for days, not merely following it, but wheeling in wide circles around it without ever being observed to land on the water." - Wikipedia

Data:

13:00   14:00   15:00   16:00   17:00

Wet at least once   Wet at least once

Dry for full hour
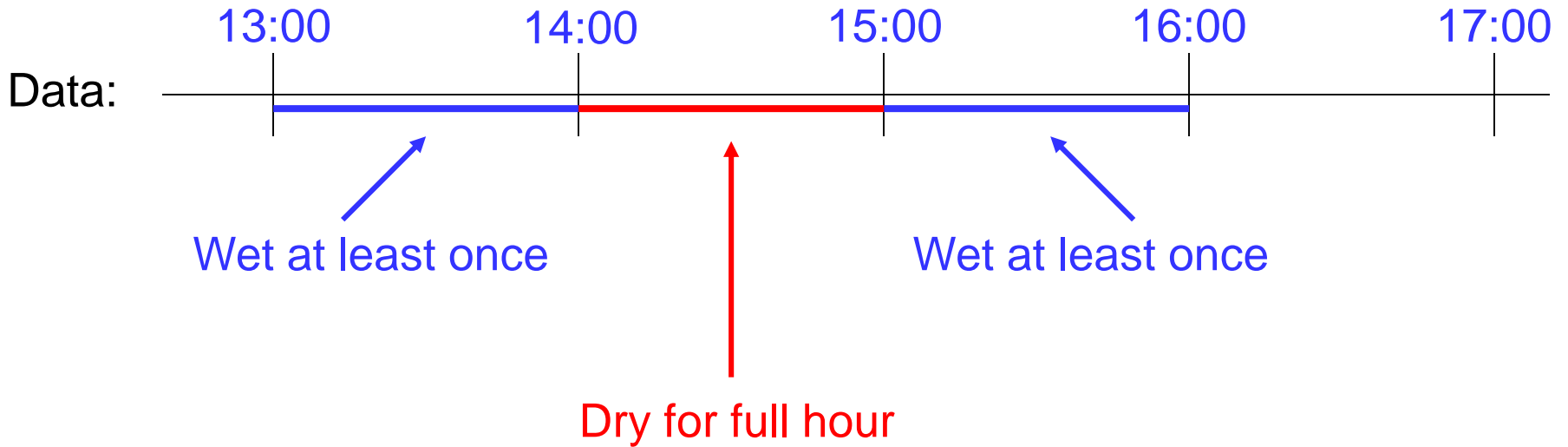
Time series data is number of consecutive hours that bird is not wet.

Levy Environmental Hypothesis I
V96

Having established the existence of scale invariance, we now turn to the question of its origin. Although scale invariance is widely observed in biology[2,10–18], the basis for such scale-invariant behaviour has remained elusive. Scale invariance in complex systems could be caused by nonlinear dynamics, as it is well known that nonlinear dynamics can give rise to intermittency, chaos and scale invariance[19]. However, it has been speculated that scale invariance may confer biological advantages related to adaptability of response; for example, loss of scale invariance for heartbeat intervals corresponds to a diseased state[2,12]. Scale invariance in foraging patterns may reflect the exploitation of highly complex environments which might themselves have fractal properties.[20–22]

Levy Environmental Hypothesis II
V96
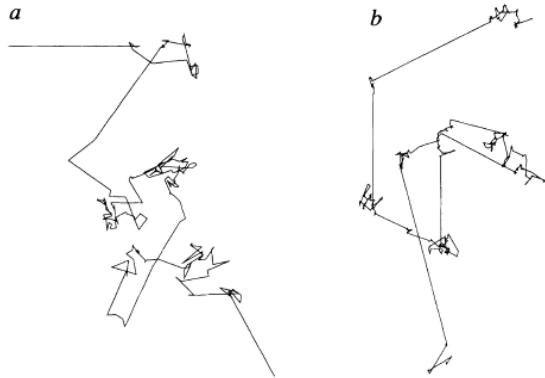
temporal behaviour to a possible spatial scale-invariance property in the underlying ecosystem[20]. It is known that the points visited by a Lévy flight form a fractal with dimension $D = \mu - 1$ (refs 2, 23). Figure 4 illustrates typical flight patterns constructed from the data and from the model, assuming that the distance travelled is proportional to the time spent dry, and that the flight directions change randomly after spending time in water. Although the latter assumption is unrealistic, it is nevertheless equally unrealistic for both the model and for the real data. The landing points of the birds have spatially scale-invariant properties, which may indicate that the distribution of food on the ocean surface is also scale invariant[27]. If this is so, then there would be voids on all length scales where there is little or no food, and birds that fail to produce a scale-invariant distribution of flight-time intervals would face a greater difficulty finding food, and hence surviving. It is also not inconceivable that the power-law distribution of flight intervals may be related to the lifetime distribution of the thermals used by the birds to fly[1].

FIG. 4 a, Possible flight path of a bird constructed from the longest time series, as described in the text. The time resolution of the data prevents us from considering changes in flight directions which occur more frequently than once per hour. b, Possible flight path given by the Lévy-walk model discussed in the text. Both flight paths have scale-invariant 'fractal' properties which may indicate that the distribution of food on the ocean surface is spatially scale invariant.

Lévy Foraging Hypothesis I

Schlesinger & Klafter ended their comparison of Lévy walks & Lévy flights in a landmark 1985 NATO ASI volume with this comment: " It has been suggested [B. Ninham, priv. comm.] that certain animals such as ants perform Levy walks when searching for food in a new area. The above analysis may imply that starving Levy walk ants possess a slight evolutionary advantage over ants performing other walks, such as even the [self avoiding walk]. Flying ants can be considered by the reader." – in "On Growth & Form", Stanley & Ostrowsky (eds.) Editions Nijhoff, 1986.

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Lévy Foraging Hypothesis II

An additional potential advantage of Lévy flights, suggested first for insects (M. F. Shlesinger, personal communication), relates to foragers that operate in swarms or flocks. After t steps, a single brownian walker in two dimensions visits $t/\ln t$ new sites, whereas a single Lévy walker visits t new sites[28]. This is not a large difference: Lévy flights are not much better than brownian motion in terms of reaching new sites. But for a 'swarm' comprised of N walkers, there is a large difference between the brownian walk and the Lévy flight: after t steps, a brownian swarm of N walkers visits only $t\ln(N/\ln t)$ distinct sites until an astronomically large crossover time $t_x \sim e^N$ is reached[28], whereas the swarm of Lévy walkers visits Nt sites. Thus the Lévy flight pattern allows the individual to visit new sites that the swarm has not visited. As some prey can migrate vertically in response to predators, the food at a given site may become unavailable for some time, thereby forcing the birds to find new sites not yet visited. Thus Lévy flight search patterns in animal behaviour may reflect the solution of the biological search problem in complex environments, and it would be interesting to repeat this type of study for birds whose

V96

Lévy Foraging Hypothesis II

An additional potential advantage of Lévy flights, suggested first for insects (M. F. Shlesinger, personal communication), relates to foragers that operate in swarms or flocks. After t steps, a single brownian walker in two dimensions visits $t/\ln t$ new sites, whereas a single Lévy walker visits t new sites[28]. This is not a large difference: Lévy flights are not much better than brownian motion in terms of reaching new sites. But for a 'swarm' comprised of N walkers, there is a large difference between the brownian walk and the Lévy flight: after t steps, a brownian swarm of N walkers visits only $t\ln(N/\ln t)$ distinct sites until an astronomically large crossover time $t_x \sim e^N$ is reached[28], whereas the swarm of Lévy walkers visits Nt sites. Thus the Lévy flight pattern allows the individual to visit new sites that the swarm has not visited. As some prey can migrate vertically in response to predators, the food at a given site may become unavailable for some time, thereby forcing the birds to find new sites not yet visited. Thus Lévy flight search patterns in animal behaviour may reflect the solution of the biological search problem in complex environments, and it would be interesting to repeat this type of study for birds whose

V96

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Optimizing the success of random searches

G. M. Viswanathan[*†‡]**, Sergey V. Buldyrev**[*]**, Shlomo Havlin**[*§]**, M. G. E. da Luz**[‖¶]**, E. P. Raposo**[‖#] **& H. Eugene Stanley**[*]

* Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215, USA
† International Center for Complex Systems and Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, 59072-970, Natal-RN, Brazil
‡ Departamento de Física, Universidade Federal de Alagoas, 57072-970, Maceió-AL, Brazil
§ Gonda-Goldschmied Center and Department of Physics, Bar Ilan University, Ramat Gan, Israel
‖ Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
¶ Departamento de Física, Universidade Federal do Paraná, 81531-970, Curitiba-PR, Brazil
# Laboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, 50670-901, Recife-PE, Brazil

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We address the general question of what is the best statistical strategy to adapt in order to search efficiently for randomly located objects ('target sites'). It is often assumed in foraging theory that the flight lengths of a forager have a characteristic scale: from this assumption gaussian, Rayleigh and other classical distributions with well-defined variances have arisen. However, such theories cannot explain the long-tailed power-law distributions[1,2] of flight lengths or flight times[3–6] that are observed experimentally. Here we study how the search efficiency depends on the probability distribution of flight lengths taken by a forager that can detect target sites only in its limited vicinity. We show that, when the target sites are sparse and can be visited any number of times, an inverse square power-law distribution of flight lengths, corresponding to Lévy flight motion, is an optimal strategy. We test the theory by analysing experimental foraging data on selected insect, mammal and bird species, and find that they are consistent with the predicted inverse square power-law distributions.

Viswanathan et al 99

Lévy Foraging Hypothesis II

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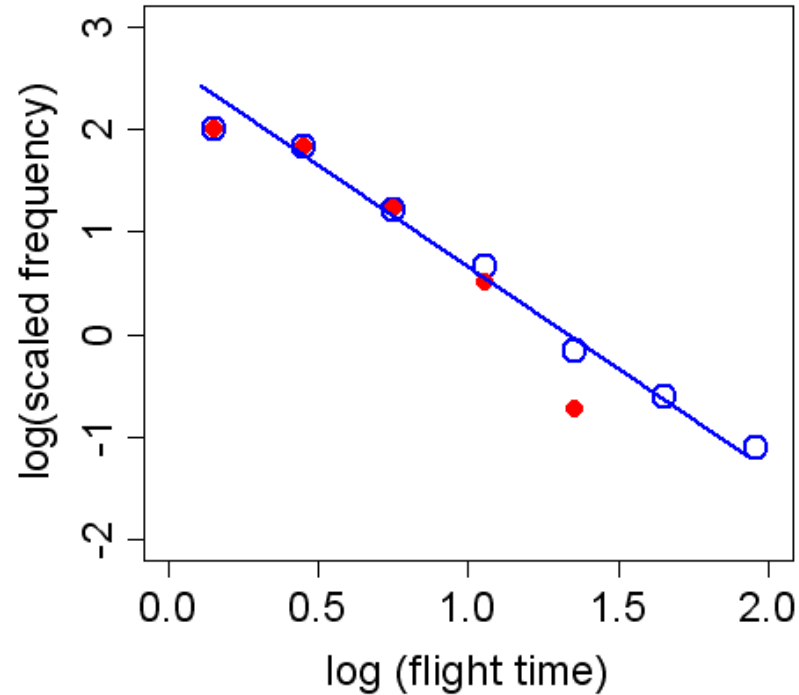
V96

Viswanathan et al 99

Edwards, Philips, Watkins et al, 2007

- Time series <u>including</u> take off and landing

Hours since logger switched on
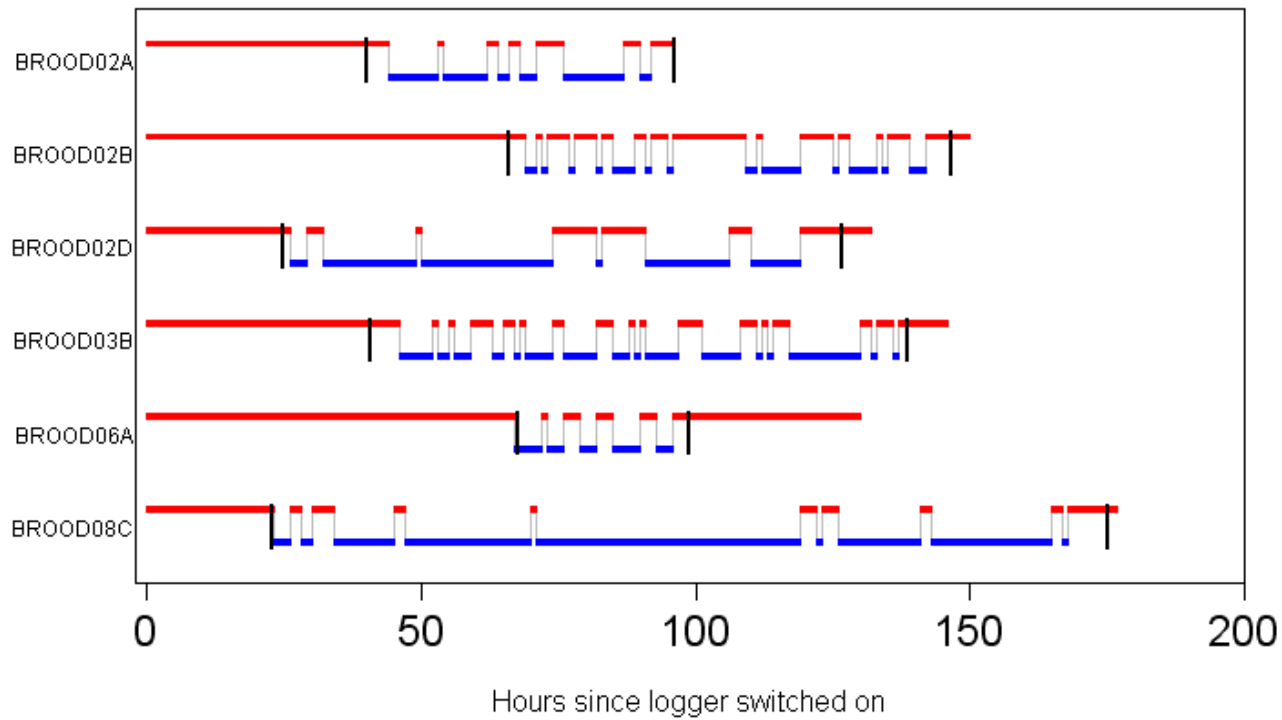
Edwards et al, op cit.

- Waiting times

  after correction:

Short-tailed, unlike a Lévy walk …but like the " studies [which] found [birds spent]… as long as 18 hours a day in flight": Poncet, Nat. Geographic, March 1989.

Re-examination of LW/LFH …

- Travis, 2007; Buchanan, 2008; Barabasi, 2010;Viswanathan et al 2012. …

NEWS**FOCUS**

ECOLOGY

Do Wandering Albatrosses Care About Math?

Repudiating a decade-old study of sea birds, a new report questions a popular model of how animals—as well as fishing boats and people—search for food

Animal behaviour is an endless challenge to mathematical modellers. In the first of two features, **Mark Buchanan** looks at how a mathematical principle from physics might be able to explain patterns of movement. In the second, **Arran Frood** asks what current models can teach us about ecological networks half a billion years old.

THE MATHEMATICAL MIRROR TO ANIMAL NATURE

… only increased activity in the field
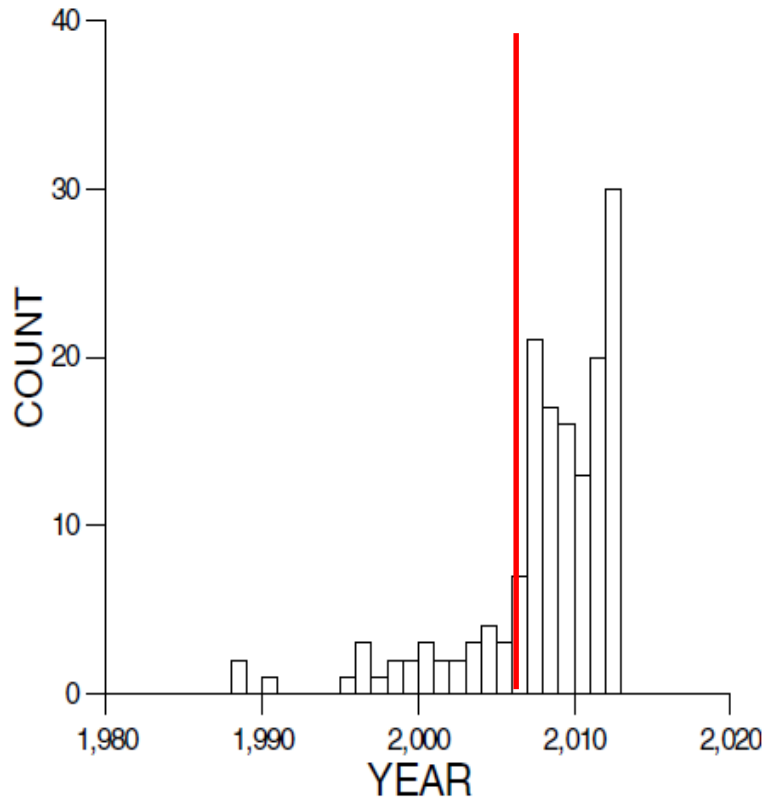
Two particular legacies of time:

Renewed attention to statistical inference and other issues around measurement

Consideration of a wider range of models and paradigms

Figure 1: Numbers of published articles per year (through 2012) that consider Lévy walks and movements by organisms **Pyke, 2014**

Gedanken experiment

How would cleverest <u>random walker</u> play battleships
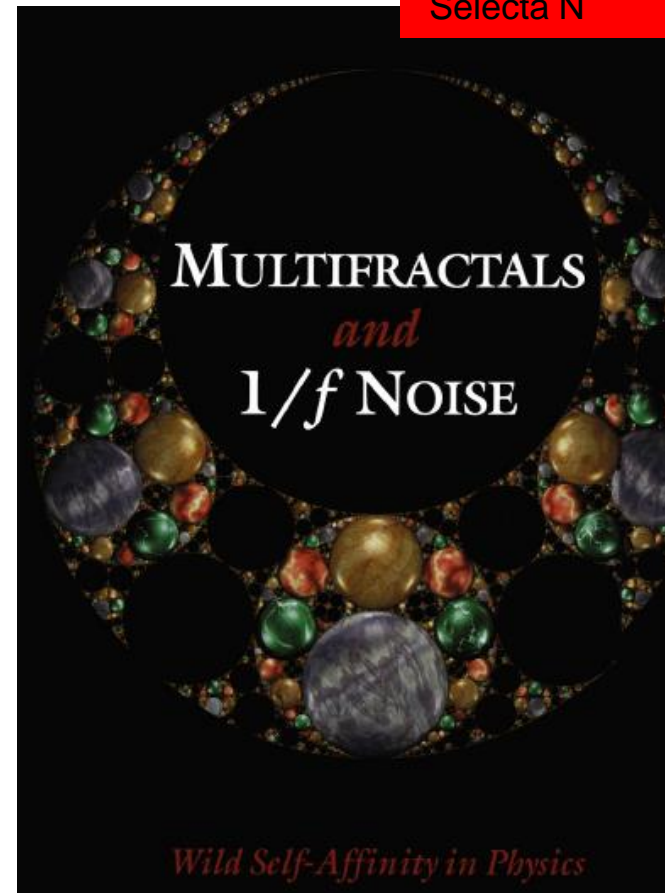
How does even <u>young child</u> play battleships ?

MY INTERESTS

Models for "1/f"

Selecta H
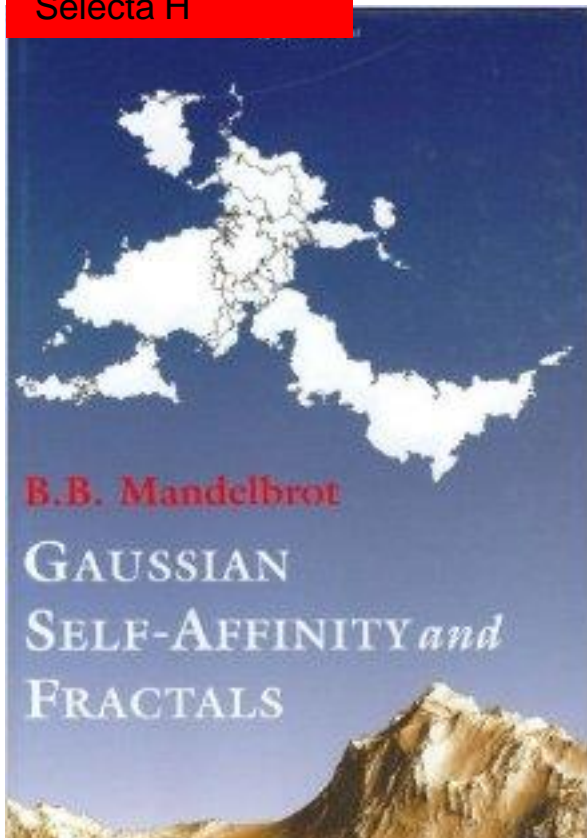
Selecta N

My models of both telephone errors and Nile floods involved spectra of the form f^{-B}. Despite this common property, those processes were of totally different character. That is, a common spectrum did not imply any deeper commonality.

Choices for time dependence

- CTRW is Markovian in structure- "memory" is modelled by using heavy tailed times between jumps. **[Mandelbrot, 65-67] knew** that FTP-like process was non-ergodic because its periodogram grows with observation time !

- Alternative is an ergodic, non-Markovian process where memory is embodied in the kernel **[Mandelbrot and co-authors, 65-68]**

Some Noises with $1/f$ Spectrum, a Bridge Between Direct Current and White Noise

BENOIT MANDELBROT, SENIOR MEMBER, IEEE
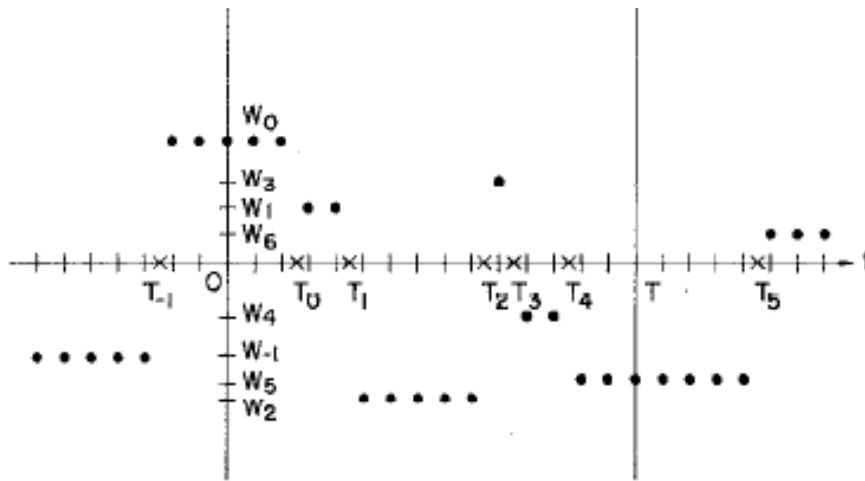
Fig. 2.

"What you see is what you wait for"

Fig. 3. This is a representation of the positive portions of Paul Lévy's symmetric stable distributions, with $1 \leq \theta \leq 2$. Abscissa: $\log u$; ordinate: $\log \Pr(U > u)$. The limit cases $\theta = 2$ and $\theta = 1$ are the classical laws of Gauss and of Cauchy. Note that, if u is large enough, one has $\Pr(U > u) \sim (u/u_a^*)^{-\theta}$.

- Abrupt state changes
- Fat distributions of switching times: "Levy" ($E[t^2] = \infty$) case.

The conditional spectrum:

follows. A) Gaussianity is a very subtle notion (See Chapter N4). B) In an environment that is not exactly Gaussian, a numerical spectrum found by experimentalists to be proportional to $1/f^B$ *need not* be an estimate of an underlying Wiener-Khinchin spectrum. It may well be a more generally valid expression that is described in M1967b{N10} and called *conditional spectrum*. In addition to f, a conditional spectral density $S(f)$ depends on a conditioning length T and takes the form $TG(Tf)$, where $0 < G(0) < \infty$, but $G(f) \sim f^{-B}$ for $f \to \infty$.
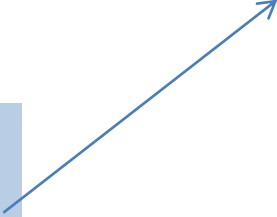
Mandelbrot 1967 reviewed in N2, Selecta, 1999

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- "*Numerical … 1/f … spectrum … need not … estimate … Wiener-Khinchine spectrum*". Instead "*depends on conditioning length T*". Unlike stationary LRD model, singularity is an artefact.

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Mandelbrot 1967 reviewed in N2, Selecta, 1999

fBm

FRACTIONAL BROWNIAN MOTIONS, FRACTIONAL NOISES AND APPLICATIONS*

BENOIT B. MANDELBROT† AND JOHN W. VAN NESS‡

The basic feature of fBm's is that the "span of interdependence" between their increments can be said to be infinite. By way of contrast, the study of

$$\text{fBm: } X_{H,2}(t) \sim \int_R \left[(t-s)_+^{H-\frac{1}{2}} - (-s)_+^{H-\frac{1}{2}} \right] dL_2(s)$$

Infinite range memory kernel

Gaussian

What does fBm mean ?

*[…], **if infinite dependence is necessary it does not mean that IBM's details of ten years ago influence IBM today, because there's no mechanism within IBM for this dependence.** However, IBM is not alone. The River Nile is [not] alone. **They're just one-dimensional corners of immensely big systems**. The behaviour of IBM stock ten years ago does not influence its stock today through IBM, but IBM the enormous corporation has changed the environment very strongly. The way its price varied, went up or went up and fluctuated, had discontinuities, had effects upon all kinds of other quantities, and they in turn affect us. –*
Mandelbrot, interviewed in 1998 by B. Sapoval for Web of Stories

What does fBm mean ?

*[…], **if infinite dependence is necessary it does not mean that IBM's details of ten years ago influence IBM today, because there's no mechanism within IBM for this dependence.** However, IBM is not alone. The River Nile is [not] alone. **They're just one-dimensional corners of***

- Resolution of apparent paradox is that world as a whole is Markovian, the memory is a consequence of looking at a piece of it. Generalises the Mori-Zwanzig idea.

very strongly. The way its price varied, went up or went up and fluctuated, had discontinuities, had effects upon all kinds of other quantities, and they in turn affect us. –
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Conclusions

- Random walks and biology co-evolved from earliest days

- Still a productive dialogue to which ASG will contribute

- Also links to deep and current issues in fundamental statistical mechanics such as weak ergodicity breaking

SPARES

Theme

Hurst
effect

Will today distinguish three things often taken as same

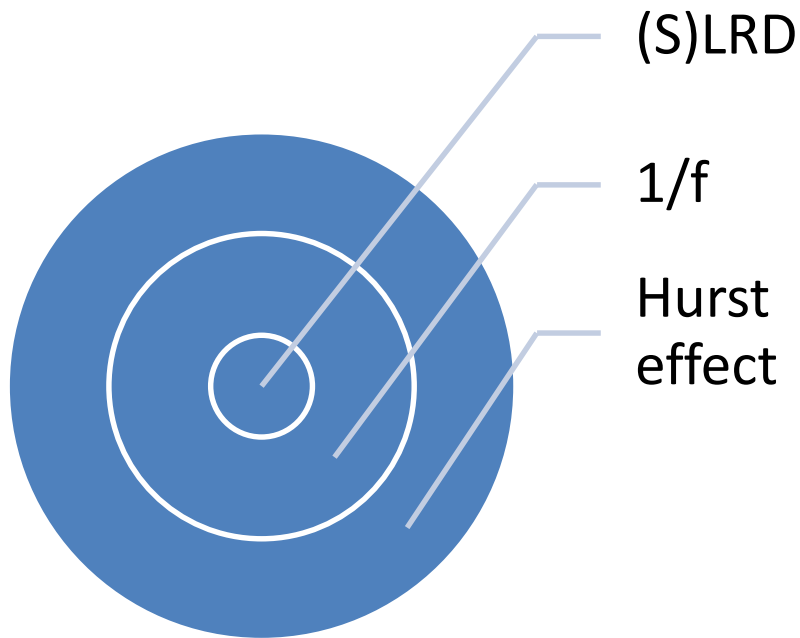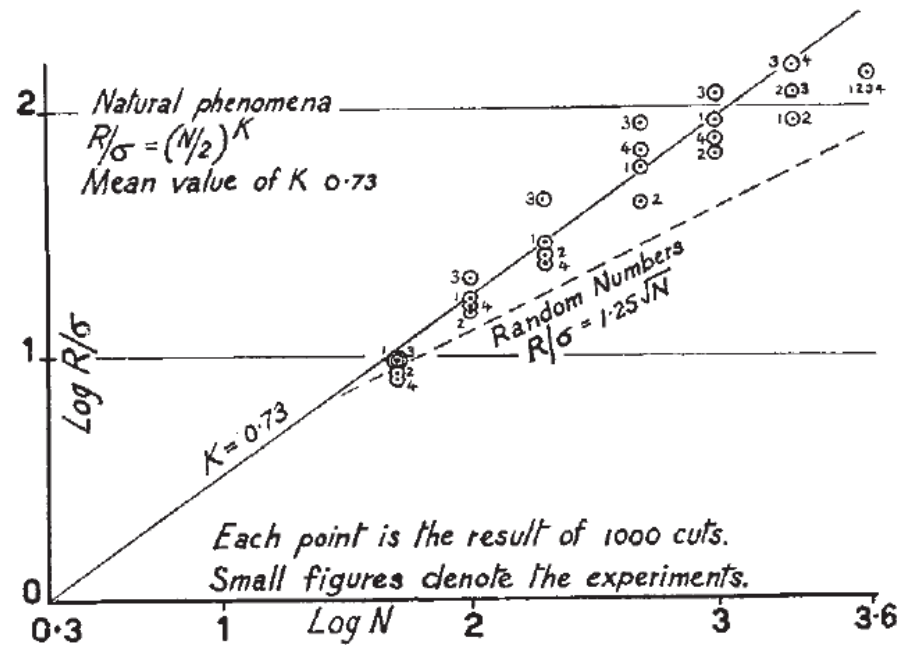- Observed growth of range in time series: "Hurst effect"
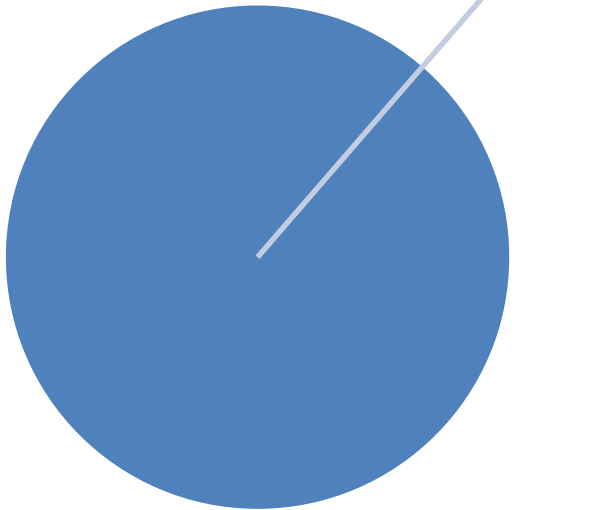
Theme
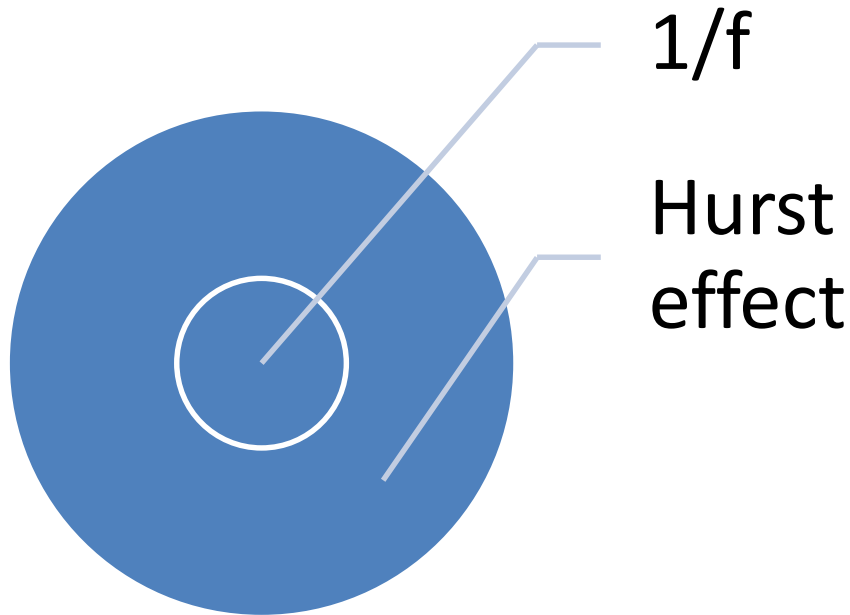
1/f

Hurst
effect

Will today distinguish three things often taken as same

- Observed growth of range in time series: "Hurst effect"

- Observation of a singularity at zero in Fourier spectra: "1/f"

Theme

(S)LRD

1/f

Hurst effect

Will today distinguish three things often taken as same

- Observed growth of range in time series: "Hurst effect"
- Observation of a singularity at zero in Fourier spectra: "1/f"
- The long range dependence seen in stationary 1/f case: (S)LRD.
- Using 1/f as a diagnostic of LRD <u>assumes</u> stationarity

Fact: Anomalous growth of range

Hurst
Effect

Natural phenomena

$$R/\sigma = (N/2)^K$$

Mean value of K 0·73

$K = 0.73$

Random Numbers $R/\sigma = 1.25\sqrt{N}$

Each point is the result of 1000 cuts.
Small figures denote the experiments.

Log R/σ

Log N

"I heard about the … Nile … in '64, ... the variance doesn't draw
time span as you take bigger and bigger integration intervals;
it goes like time to a certain power different from one. … Hurst …
was getting results that were incomprehensible". – ***Mandelbrot,***

Formula: Long Range Dependence

(S)LRD
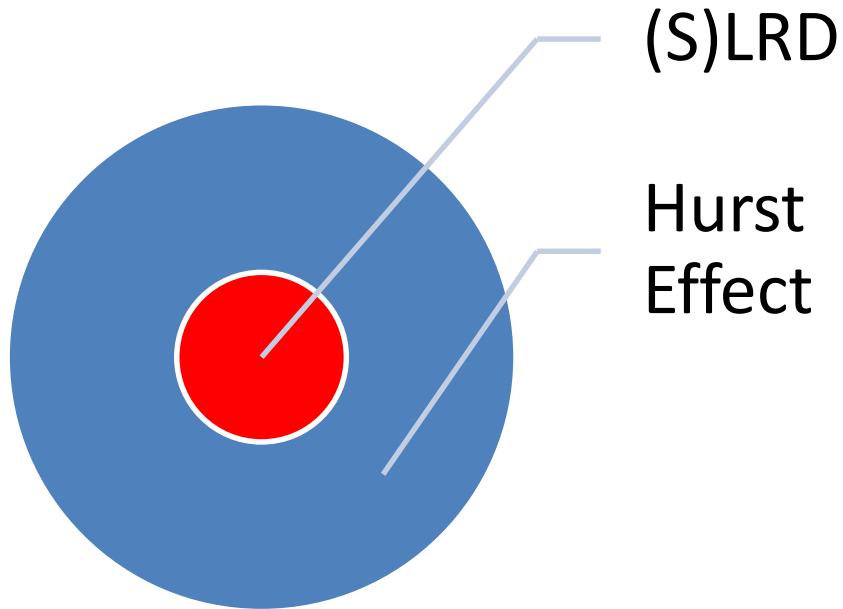
Hurst
Effect

- **Mandelbrot, van Ness, and Wallis, 1965-69**
- First **[history in Graves et al, arXiv, 2014a]** demonstration that Hurst effect could be explained by stationary long range dependent process
- Model, fractional Gaussian noise [see also **Kolmogorov's** "Wiener Spiral"], had singular spectral density at lowest frequencies.

$$S'(f) \sim f^{-\beta}$$

The 1/f "paradox"

If spectral density $S'(f) \sim f^{-\beta}$

then i) it is singular as $f \to 0$

and ii) if we define an autocorrelation
function via $\rho(\tau) = <x(t)x(t+\tau)>$
and use Wiener-Khinchine theorem to
get ρ from Fourier transform of $S'(f)$

then ρ falls off as power law, and its
summed lags "blow up" $\sum_\tau \rho(\tau) \to \infty$

Fractional motions and noises
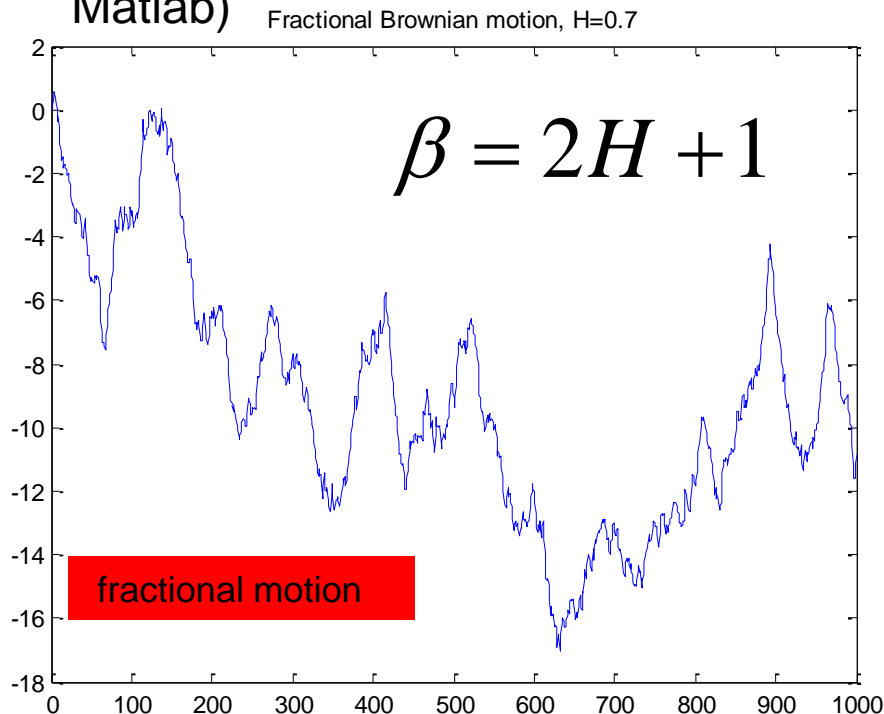
$$\text{fBm: } X_{H,2}(t) \sim \int_R \left((t-s)_+^{H-\frac{1}{2}} - (-s)_+^{H-\frac{1}{2}} \right) dL_2(s)$$

Build a nonstationary, self similar walk … (used wfbm in Matlab)

Fractional Brownian motion, H=0.7

$$\beta = 2H + 1$$

fractional motion

Fractional motions and noises

$$\text{fBm: } X_{H,2}(t) \sim \int_R \left((t-s)_+^{H-\frac{1}{2}} - (-s)_+^{H-\frac{1}{2}} \right) dL_2(s)$$

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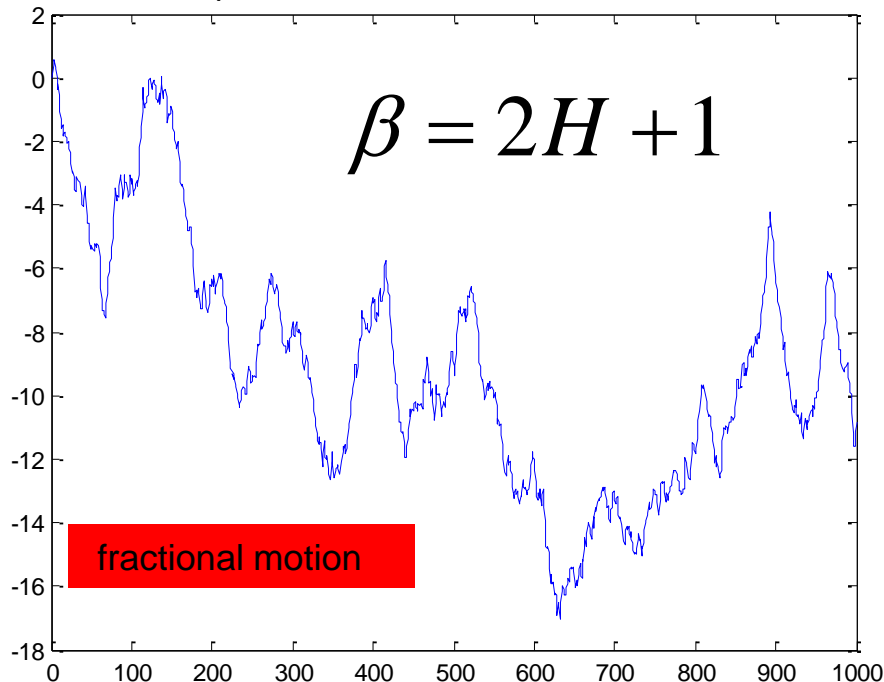
Then differentiate to give a stationary LRD noise

Fractional Brownian motion, H=0.7

$$\beta = 2H + 1$$

fractional motion

Fractional Gaussian noise

$$\beta = 2H - 1$$

fractional noise

1/f without (S)LRD

(S)LRD

1/f

Hurst
effect
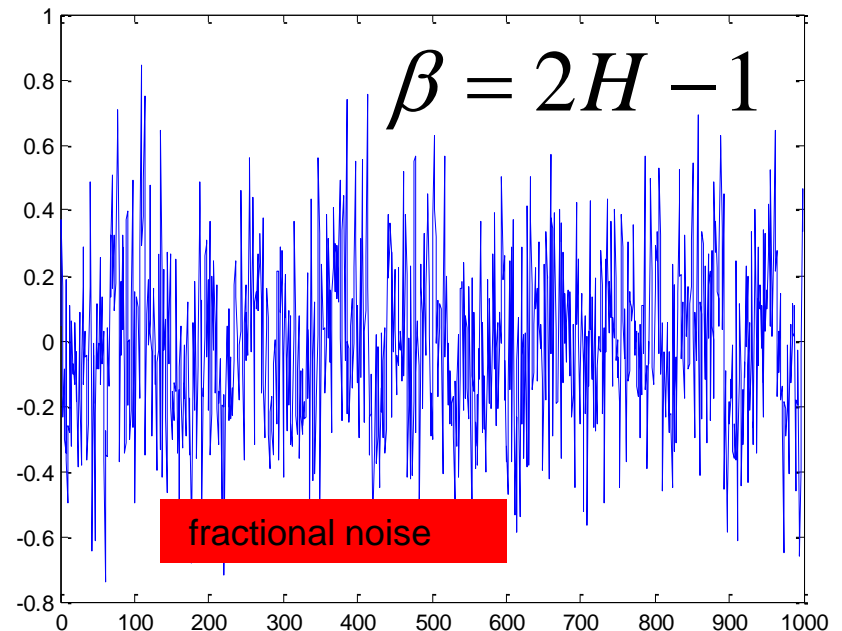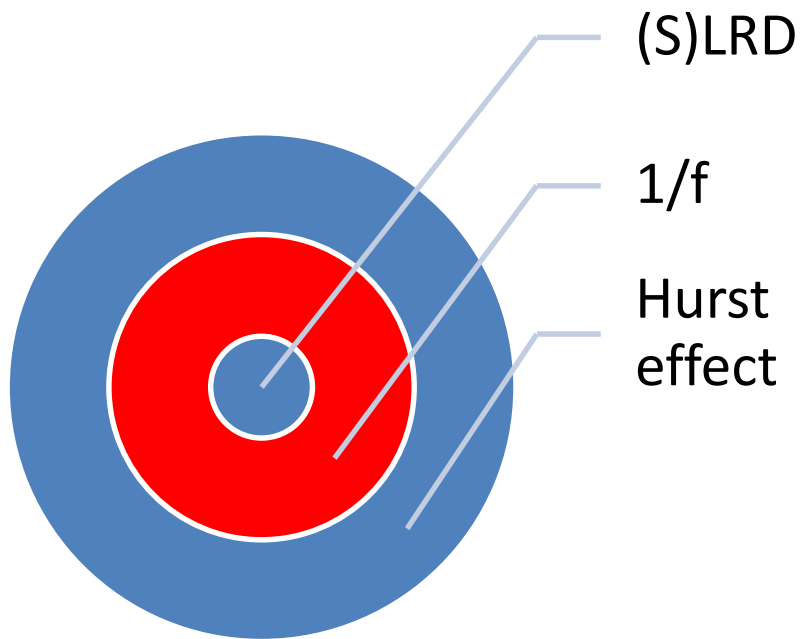
- Before (S)LRD models, **Mandelbrot [1963-67]** had proposed other 1/f models which were not stationary LRD in same sense as fGn.

- Solved 1/f paradox by a different route. Still little known in the geosciences [but see **Klemes, WRR, 1974**].

Formula versus fact
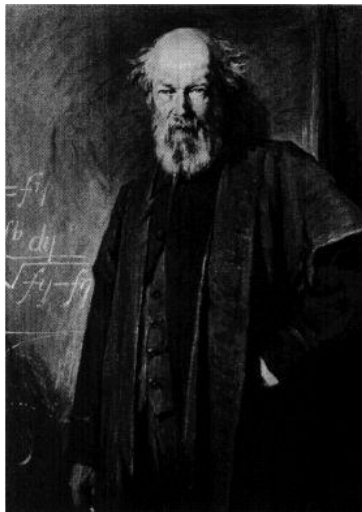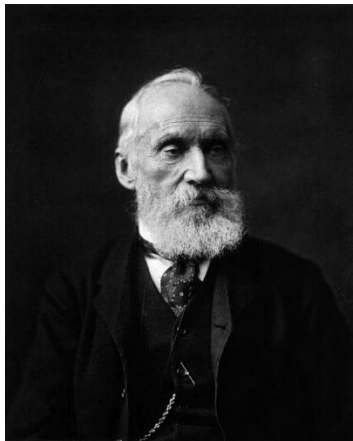
"*Nothing can be more fatal to progress than a too confident reliance on mathematical symbols; for the student is only too apt to … **consider the formula and not the fact as the physical reality**"*.
Thomson (Kelvin) & Tait, 1890 edition.

"*Like the ear, **the eye is very sensitive to features that the spectrum does not reflect**. Seen side by side, different 1/f noises, Gaussian [i.e. fGn], dustborne [i.e. fractional renewal] and multifractal, **obviously** differ from one another*"-
Mandelbrot, Selecta N, 1999.

Open (L) v closed (R)Brownian walks
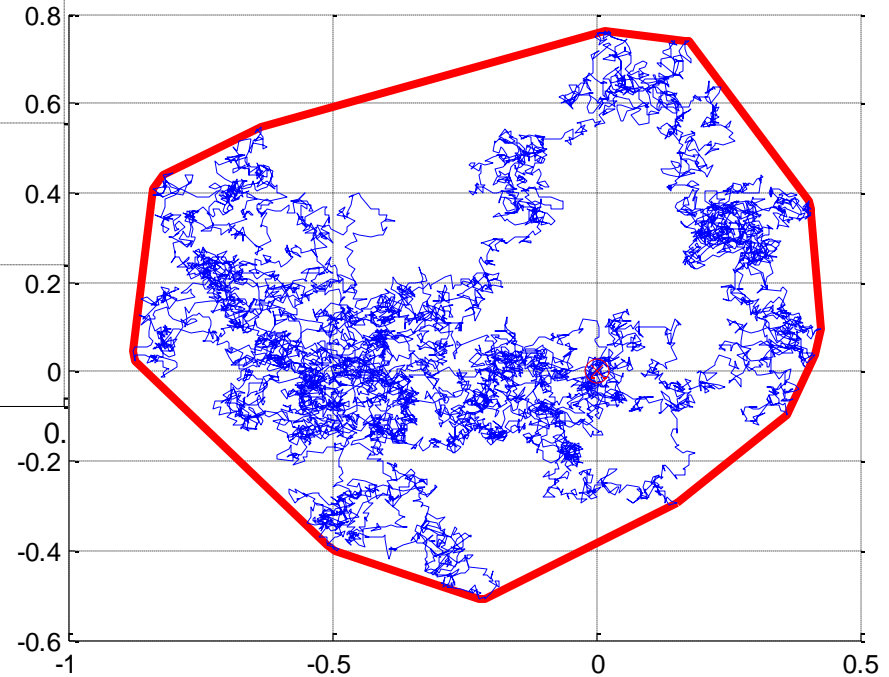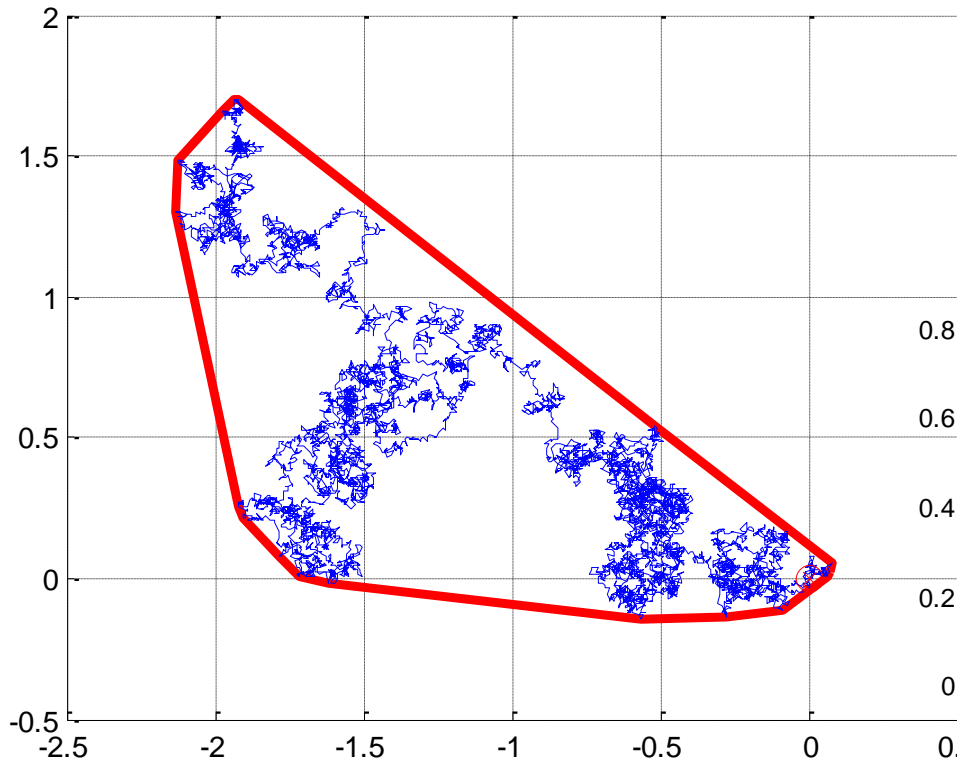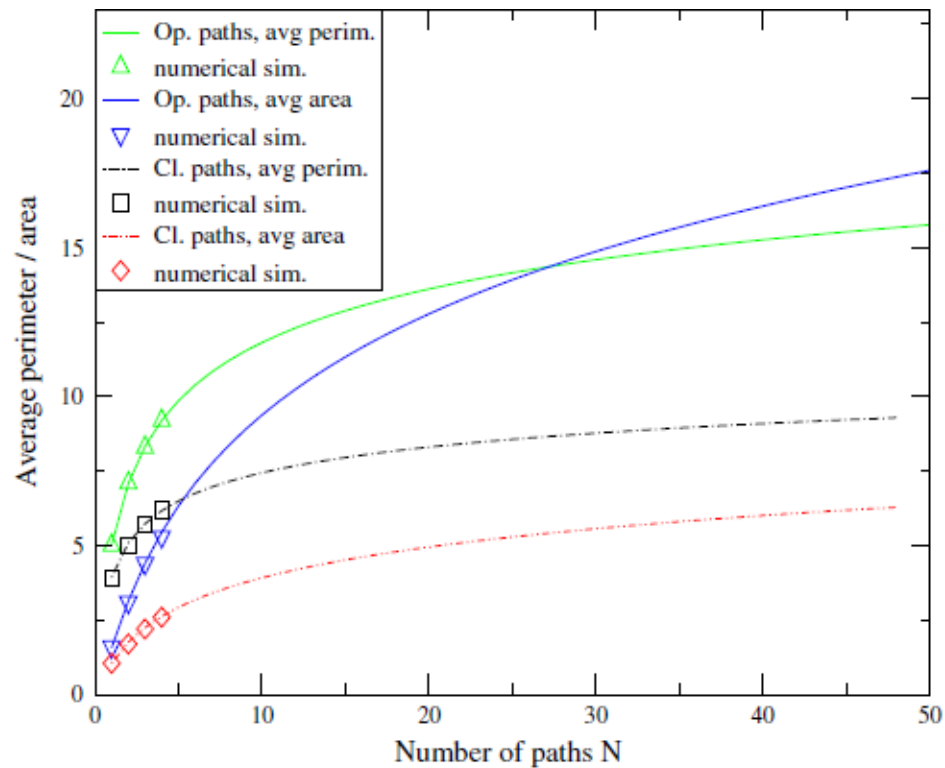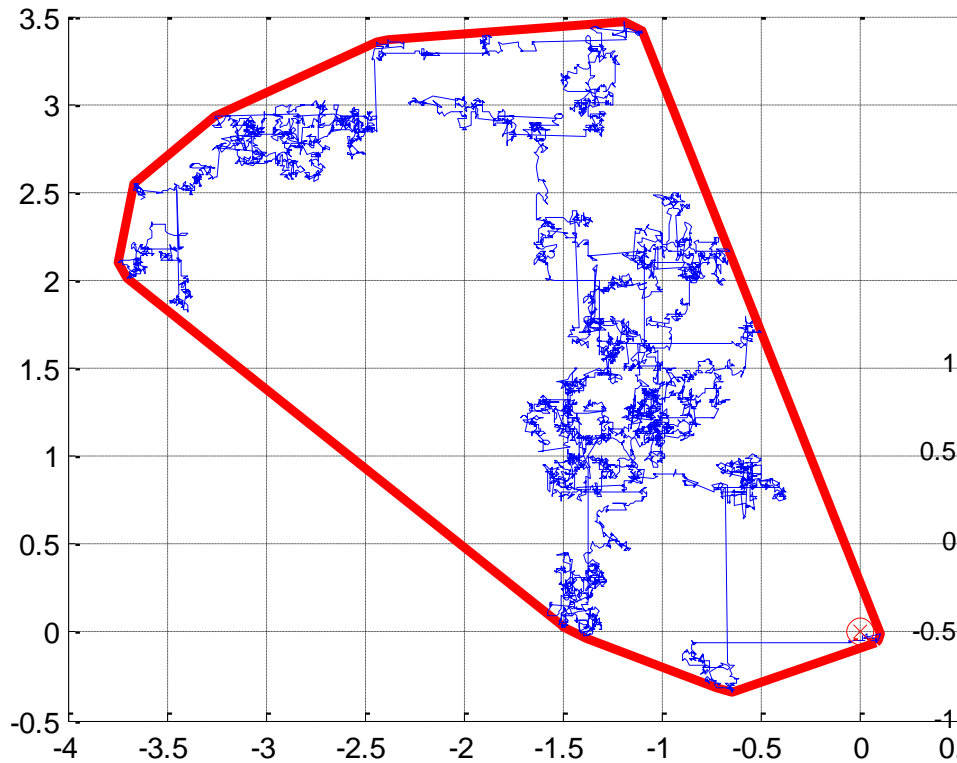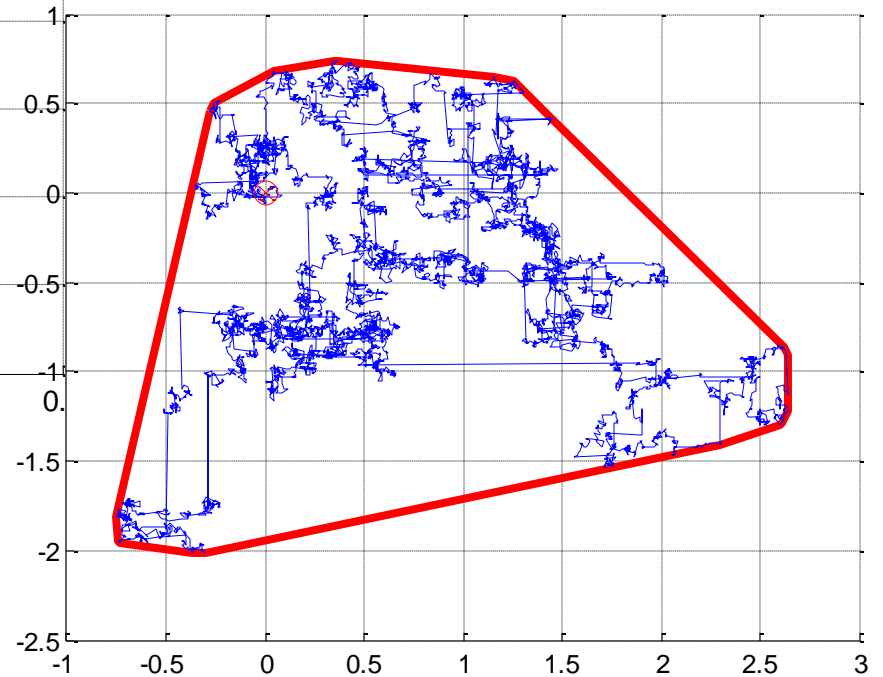

α=2

Randon-Furling et al, 2009

FIG. 4 (color online). Setting $T = 1$, the analytical results for average perimeter α_N [Eq. (7)] and area β_N [Eq. (9)] of N open (Op.) Brownian paths, and similarly the average perimeter $\alpha_N(c)$ [Eq. (10)] and area $\beta_N(c)$ [Eq. (11)] of N closed (Cl.) Brownian paths, plotted against N. The symbols denote results from numerical simulations (up to $N = 4$).

Open (L) v closed (R) Lévy flights

α=1.6

Boyer et al (2008)

- Boyer et al, arXiv, 2008 pointed out that jump sizes of foraging model in Viswanathan et al, 1999 not same as those of simple Levy walk used in Viswanathan et al, 1996. Worked out distribution explicitly:

$$P(x) = P_0(x)\, e^{-x/\tau} + \tau^{-1}\, e^{-x/\tau} \int_x^\infty du P_0(u) \qquad (1)$$

with $P_0(x) = C x^{-\mu}$ ($\mu > 1$) being the *choice* distribution, and C a normalization constant. The first term in (1) is the probability of making a trip of duration x and not finding anything, the second the probability of finding a target after a time x (implying that the chosen time u is $>x$). P takes two limiting forms: *(i)* $P(x) \approx C x^{-\mu}$ if $x \ll \tau(\mu-1)$, a wide interval only if resources are scarce; *(ii)* $P(x) \sim x^{1-\mu} e^{-x/\tau} \neq P_0(x)$ if $x \gg \tau(\mu-1)$.